Behavioral and Structural Barriers to Information Aggregation in Networks
Theoretical Appendix

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A Brief Discussion of the Assumptions of the Myopic Bayesian Model

Here, we discuss four implications of the myopic Bayesian model's assumptions. First, assuming myopia is restrictive.[1] In some network structures and in some signals' distributions, it may be dynamically optimal to take actions that are not optimal from a myopic perspective.[2] Second, due to the myopia assumption, the model describes a decision problem rather than a strategic game. This is because there are no payoff externalities: each agent's preferences depend only on her own actions and the distribution of initial signals. Without myopia, dynamic considerations could turn the setting into a game of strategic information revelation. Third, the model implies that each agent believes others act according to myopic Bayesian reasoning. This assumption may be violated in experimental settings, where subjects might identify behavior inconsistent with Bayesian inference.[3] Finally, agents hold no prior over the behavior of others who are indifferent between actions based on their histories (see Footnote 37 in the main text). As a result, in some networks and for some signals' distributions, the model does not yield a unique prediction.

B Formal Statements and Proofs

Lemma 1

Lemma 1. *For every $i \in N$: If $s(i) = w$ then $a_i^1 = W$ and if $s(i) = b$ then $a_i^1 = B$.*

Proof. For deciding the first period's action note that (i) the history is irrelevant and therefore (ii) G is irrelevant.

[1] Myopia is typically justified either by assuming each node represents a continuum of agents (e.g., Gale and Kariv (2003)), or by imposing strong discounting.

[2] For example, agent i might wish to study how her actions influence those of her neighbors, using these reactions to better infer their information and beliefs.

[3] For example, selecting an action that contradicts the majority in the Complete network, or switching actions in the absence of new information. See Chandrasekhar et al. (2020) for a related discussion of mixed models in which some agents are myopic Bayesian and others are naïve.

Suppose $s(i) = w$. Agent i updates her belief regarding the state of the world:

$$P(\omega = WHITE|s(i) = w) = \frac{0.5q}{0.5q + 0.5(1-q)} = \frac{0.5q}{0.5} = q$$

Next, she updates her belief regarding the majority of signals.

$$P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}| + 1 > |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|s(i) = w\Big) =$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}|+1 > |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|\omega = WHITE\Big)\times P\Big(\omega = WHITE\Big|s(i) = w\Big)+$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}| + 1 > |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|\omega = BLUE\Big) \times P\Big(\omega = BLUE\Big|s(i) = w\Big)$$

First, suppose that n is even:

$$= \Bigg[\sum_{k=\frac{n}{2}}^{n-1} \binom{n-1}{k} q^k(1-q)^{n-k-1}\Bigg] \times q + \Bigg[\sum_{k=\frac{n}{2}}^{n-1} \binom{n-1}{k} (1-q)^k q^{n-k-1}\Bigg] \times (1-q) =$$

$$= \sum_{k=\frac{n}{2}}^{n-1} \binom{n-1}{k} \Big[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1}q^{n-k-1}\Big] = \sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \Big[q^k(1-q)^{n-k} + (1-q)^k q^{n-k}\Big] >$$

$$\sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \Big[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1}q^{n-k-1}\Big] =$$ [4]

$$= \Bigg[\sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} q^k(1-q)^{n-k-1}\Bigg] \times q + \Bigg[\sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} (1-q)^k q^{n-k-1}\Bigg] \times (1-q) =$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}|+1 \leq |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|\omega = WHITE\Big)\times P\Big(\omega = WHITE\Big|s(i) = w\Big)+$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}|+1 \leq |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|\omega = BLUE\Big)\times P\Big(\omega = BLUE\Big|s(i) = w\Big) =$$

$$= P\Big(|\{j \in N\backslash\{i\}|s(j) = w\}| + 1 \leq |\{j \in N\backslash\{i\}|s(j) = b\}|\Big|s(i) = w\Big)$$

[4] To see this inequality, note that for every $k \leq \frac{n}{2} - 1$ we get that $2k+1-n < 0$. Since $1 > q > \frac{1}{2} > 1-q > 0$ we get that $q^{2k+1-n} < (1-q)^{2k+1-n}$ or $q^k(1-q)^{n-k-1} < q^{n-k-1}(1-q)^k$. Thus, $q^k(1-q)^{n-k-1}(q-(1-q)) < q^{n-k-1}(1-q)^k(q-(1-q))$. This means that for every $k \leq \frac{n}{2} - 1$ we get $q^{k+1}(1-q)^{n-k-1} - q^k(1-q)^{n-k} < q^{n-k}(1-q)^k - q^{n-k-1}(1-q)^{k+1}$. Switching sides we get that for every $k \leq \frac{n}{2} - 1$: $q^{k+1}(1-q)^{n-k-1} + q^{n-k-1}(1-q)^{k+1} < q^{n-k}(1-q)^k + q^k(1-q)^{n-k}$. Hence, $\sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \Big[q^k(1-q)^{n-k} + (1-q)^k q^{n-k}\Big] > \sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \Big[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1}q^{n-k-1}\Big]$.

Now, suppose that n is odd:

$$= \Bigg[\sum_{k=\frac{n-1}{2}}^{n-1} \binom{n-1}{k} q^k(1-q)^{n-k-1}\Bigg] \times q + \Bigg[\sum_{k=\frac{n-1}{2}}^{n-1} \binom{n-1}{k} (1-q)^k q^{n-k-1}\Bigg] \times (1-q) =$$

$$= \sum_{k=\frac{n-1}{2}}^{n-1} \binom{n-1}{k} \Bigg[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1} q^{n-k-1}\Bigg] >$$

$$> \sum_{k=\frac{n+1}{2}}^{n-1} \binom{n-1}{k} \Bigg[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1} q^{n-k-1}\Bigg] =$$

$$= \sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} \Bigg[q^k(1-q)^{n-k} + (1-q)^k q^{n-k}\Bigg] > \sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} \Bigg[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1} q^{n-k-1}\Bigg] =$$ [5]

$$= \Bigg[\sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} q^k(1-q)^{n-k-1}\Bigg] \times q + \Bigg[\sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} (1-q)^k q^{n-k-1}\Bigg] \times (1-q) =$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}|+1 < |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|\omega = WHITE\Big) \times P\Big(\omega = WHITE\Big|s(i)=w\Big) +$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}|+1 < |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|\omega = BLUE\Big) \times P\Big(\omega = BLUE\Big|s(i)=w\Big) =$$

$$= P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}| + 1 < |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|s(i)=w\Big) =$$

$$= P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}| + 1 \leq |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|s(i)=w\Big)$$

To sum up, we get that for any n,

$$P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}| + 1 > |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|s(i)=w\Big) >$$

$$P\Big(|\{j \in N\backslash\{i\}|s(j)=w\}| + 1 \leq |\{j \in N\backslash\{i\}|s(j)=b\}|\Big|s(i)=w\Big)$$

That is, the updated beliefs are such that the probability that there are strictly more w signals than b signals is greater than the probability that there are at least as many b signals as there are w signals. Thus, since the individual is a myopic utility maximizer she chooses $a_i^1 = W$. The case where $s(i) = b$ is symmetric and therefore in that case she chooses $a_i^1 = B$. $\square$

[5]To see this inequality, note that for every $k \leq \frac{n-1}{2} - 1$ we get that $2k+1-n < 0$. Since $1 > q > \frac{1}{2} > 1-q > 0$ we get that $q^{2k+1-n} < (1-q)^{2k+1-n}$ or $q^k(1-q)^{n-k-1} < q^{n-k-1}(1-q)^k$. Thus, $q^k(1-q)^{n-k-1}(q-(1-q)) < q^{n-k-1}(1-q)^k(q-(1-q))$. Therefore, for every $k \leq \frac{n-1}{2} - 1$: $q^{k+1}(1-q)^{n-k-1} - q^k(1-q)^{n-k} < q^{n-k}(1-q)^k - q^{n-k-1}(1-q)^{k+1}$. Switching sides we get that for every $k \leq \frac{n-1}{2} - 1$: $q^{k+1}(1-q)^{n-k-1} + q^{n-k-1}(1-q)^{k+1} < q^{n-k}(1-q)^k + q^k(1-q)^{n-k}$. Hence, $\sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} \Bigg[q^k(1-q)^{n-k} + (1-q)^k q^{n-k}\Bigg] > \sum_{k=0}^{\frac{n-1}{2}-1} \binom{n-1}{k} \Bigg[q^{k+1}(1-q)^{n-k-1} + (1-q)^{k+1} q^{n-k-1}\Bigg]$.

Lemma 2

In the theoretical setting (Section 4.1) agents posses perfect recall, that is, in each round $t \in \{2, \dots\}$, before making a decision, each agent can observe the choices made by herself and her direct neighbores in all previous rounds. Hence, the history agent i observes at the beginning of period $t > 1$ is $h_i^t : B(i) \cup \{i\} \times \{1, \dots, t-1\} \to \{W, B\}$. Note that h_i^t is defined starting $t = 2$ since when taking the decision on the action in period 1, the agent has no observations on herself or her neighbors' previous actions. For every $j \in B(i) \cup \{i\}$ denote by $h_i^t(-j)$ the restriction of h_i^t to $\{B(i) \cup \{i\}\} \backslash \{j\}$.

Lemma 2. *For every $i \in N$ and for every $j \in B(i)$: If $h_i^2(j, 1) = W$ then $s(j) = w$ and if $h_i^2(j, 1) = B$ then $s(j) = b$.*

Proof. Let $h_i^2(j, 1) = W$. Suppose, in contradiction that $s(j) = b$. Then, by Lemma 1, $a_j^1 = B$. Hence, by definition, $h_i^2(j, 1) = B$. Contradiction. Hence, $s(j) = w$. Note that since we assume common knowledge of myopic Bayesianism, agent i is able to make such an inference. Similarly, $h_i^2(j, 1) = B$ implies that $s(j) = b$. □

Lemma 3

Denote the set of agent i's neighbors that chose the action W in period $t-1$ by $W^{t-1}B(i) = \{j \in B(i) | h_i^t(j, t-1) = W\}$ and the set of agent i's neighbors that chose the action B in period $t-1$ by $B^{t-1}B(i) = \{j \in B(i) | h_i^t(j, t-1) = B\}$. Denote $\Delta_{t-1}(i) = |W^{t-1}B(i)| - |B^{t-1}B(i)|$. For brevity, we omit the subscript 1 of Δ in the proof.

Lemma 3. *For every $i \in N$:*

1. *Suppose $s(i) = w$. If $\Delta_1(i) \geq 0$ then $a_i^2 = W$, if $\Delta_1(i) = -1$ then $a_i^2 \in \{W, B\}$ and if $\Delta_1(i) \leq -2$ then $a_i^2 = B$.*
2. *Suppose $s(i) = b$. If $\Delta_1(i) \geq 2$ then $a_i^2 = W$, if $\Delta_1(i) = 1$ then $a_i^2 \in \{W, B\}$ and if $\Delta_1(i) \leq 0$ then $a_i^2 = B$.*

Proof. By Lemma 2, before the decision in period 2, agent i knows her own signal ($s(i)$) and her neighbors' signals ($\{s(j) | j \in B(i)\}$) and has no information regarding the signals of the other participants. Since the signals' distribution is independent of the network structure, G is irrelevant. Therefore, for the second period's decision, the agent cares only about the number of signals of each type independently of the exact position of their receivers.

Suppose $s(i) = w$. The conditional probability that the state of nature is $WHITE$ is

$$P(\omega = WHITE | s(i) = w, |W^1B(i)|, |B^1B(i)|) =$$

$$= \frac{0.5q^{|W^1B(i)|+1}(1-q)^{|B^1B(i)|}}{0.5q^{|W^1B(i)|+1}(1-q)^{|B^1B(i)|} + 0.5(1-q)^{|W^1B(i)|+1}q^{|B^1B(i)|}} =$$

$$= \frac{q^{|W^1B(i)|+1-|B^1B(i)|}}{q^{|W^1B(i)|+1-|B^1B(i)|} + (1-q)^{|W^1B(i)|+1-|B^1B(i)|}} =$$

$$= \frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} = \frac{[\frac{q}{1-q}]^{\Delta(i)+1}}{1 + [\frac{q}{1-q}]^{\Delta(i)+1}}$$

Since $1 > q > \frac{1}{2}$, when $\Delta(i) \geq -1$ we get $P(\omega = WHITE|s(i) = w, |W^1B(i)|, |B^1B(i)|) \geq \frac{1}{2}$ while when $\Delta(i) < -1$, $P(\omega = WHITE|s(i) = w, |W^1B(i)|, |B^1B(i)|) < \frac{1}{2}$.

Similarly, suppose $s(i) = b$. Then, $P(\omega = WHITE|s(i) = b, |W^1B(i)|, |B^1B(i)|) = \frac{[\frac{q}{1-q}]^{\Delta(i)-1}}{1+[\frac{q}{1-q}]^{\Delta(i)-1}}$. Thus, when $\Delta(i) \geq 1$ we get $P(\omega = WHITE|s(i) = b, |W^1B(i)|, |B^1B(i)|) \geq \frac{1}{2}$ while when $\Delta(i) < 1$, $P(\omega = WHITE|s(i) = b, |W^1B(i)|, |B^1B(i)|) < \frac{1}{2}$.

Knowing these conditional probabilities, the agent updates her belief regarding the signals' distribution in order to guess optimally in the second round.

Note that $\Delta(i) \in \{-|B(i)|, -|B(i)|+1, \ldots, |B(i)|-1, |B(i)|\}$. For some values of $\Delta(i)$ there is no need to account for the signals received by non-neighbors, while for other values, beliefs on the signals received by non-neighbors are necessary.

We begin with the case where there is no need to account for the signals received by non-neighbors. There are two such cases - (i) There are no non-neighbores and (ii) There are enough observed signals of any type to be certain about the majority of the signals in the whole network.

In case (i), if $|B(i)| = n-1$ and $s(i) = w$ then if $\Delta(i) + 1 > 0$ the agent must choose $a_i^2 = W$, if $\Delta(i) + 1 < 0$ the agent must choose $a_i^2 = B$ and otherwise $a_i^2 \in \{W, B\}$. That is, if $|B(i)| = n-1$ and $s(i) = w$ then if $\Delta(i) > -1$ the agent must choose $a_i^2 = W$, if $\Delta(i) < -1$ the agent must choose $a_i^2 = B$ and if $\Delta(i) = -1$ then $a_i^2 \in \{W, B\}$. Similarly, if $|B(i)| = n-1$ and $s(i) = b$ then if $\Delta(i) > 1$ the agent must choose $a_i^2 = W$, if $\Delta(i) < 1$ the agent must choose $a_i^2 = B$ and if $\Delta(i) = 1$ then $a_i^2 \in \{W, B\}$.

For case (ii), suppose $s(i) = w$. Note that $\frac{|B(i)|+\Delta(i)}{2} + 1$ is the number of w signals observed by the agent ($\frac{|B(i)|+\Delta(i)}{2} = |W^1B(i)|$) and $\frac{|B(i)|-\Delta(i)}{2}$ is the number of b signals observed by the agent ($\frac{|B(i)|-\Delta(i)}{2} = |B^1B(i)|$). There are enough observed signals to be certain about the majority of signals in the whole network if $\frac{|B(i)|+\Delta(i)}{2} + 1 > \frac{n}{2}$ since then more than half of all signals are w so the subject should choose $a_i^2 = W$. That is, if $\Delta(i) > n - |B(i)| - 2$, the agent must choose $a_i^2 = W$ (since $|B(i)| < n-1$, $\Delta(i)$ is at least 1). If $\frac{|B(i)|-\Delta(i)}{2} > \frac{n}{2}$ then more than half of all signals are b, therefore the subject should choose $a_i^2 = B$. That is, if $\Delta(i) < -(n - |B(i)|)$, the agent must choose $a_i^2 = B$ (since $|B(i)| < n-1$, $\Delta(i)$ is at most -3).

Similarly, suppose $s(i) = b$. Note that $\frac{|B(i)|+\Delta(i)}{2}$ is the number of w signals observed by the agent and $\frac{|B(i)|-\Delta(i)}{2} + 1$ is the number of b signals observed by the agent. If $\frac{|B(i)|+\Delta(i)}{2} > \frac{n}{2}$ then more than half of all signals are w, therefore the subject should choose $a_i^2 = W$. That is, if $\Delta(i) > n - |B(i)|$, the agent must choose $a_i^2 = W$ (since $|B(i)| < n-1$, $\Delta(i)$ is at least 3). If $\frac{|B(i)|-\Delta(i)}{2} + 1 > \frac{n}{2}$ then more than half of all signals are b, therefore the subject should choose $a_i^2 = B$. That is, if $\Delta(i) < -(n - |B(i)| - 2)$, the agent must choose $a_i^2 = B$ (since $|B(i)| < n-1$, $\Delta(i)$ is at most -1).

We move to the case where one needs to account for the signals received by non-neighbors.

If $s(i) = w$ and $0 < |B(i)| < n-1$, only $\Delta(i) \in \{ \max\{-|B(i)|, -(n-|B(i)|)\}, \ldots, \min\{|B(i)|, n-|B(i)|-2\}\}$ (a non-empty set) requires to account for the signals received by non-neighbours. In this case, the probability that there are at least as many w signals as there are b signals is:

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \geq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| s(i) = w, \Delta(i), |B(i)|\Big) =$$

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \geq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| \omega = WHITE\Big) \times$$

$$\times P\Big(\omega = WHITE \Big| s(i) = w, \Delta(i), |B(i)|\Big) +$$

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \geq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| \omega = BLUE\Big) \times$$

$$\times P\Big(\omega = BLUE \Big| s(i) = w, \Delta(i), |B(i)|\Big)$$

The probability that there are at least as many b signals as there are w signals is:

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \leq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| s(i) = w, \Delta(i), |B(i)|\Big) =$$

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \leq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| \omega = WHITE\Big) \times$$

$$\times P\Big(\omega = WHITE \Big| s(i) = w, \Delta(i), |B(i)|\Big) +$$

$$P\Big(|\{j \in N | j \notin B(i) \cup \{i\}, s(j) = w\}| + \Delta(i) + 1 \leq |\{j \in N | j \notin B(i) \cup \{i\}, s(j) = b\}| \Big| \omega = BLUE\Big) \times$$

$$\times P\Big(\omega = BLUE \Big| s(i) = w, \Delta(i), |B(i)|\Big)$$

We study the difference between these two probabilities, first for the case that n is even and then for the case that n is odd.

Suppose that n is even and note that $|B(i)|$ and $\Delta(i)$ are even or odd together and therefore their sum and difference are always even. Note that the summations are not empty since we consider only those $\Delta(i)$s that require beliefs on the non-neighbors.

$$\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1} \binom{n-|B(i)|-1}{k} q^k (1-q)^{n-|B(i)|-1-k} \Big] \times \frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} +$$

$$+ \Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1} \binom{n-|B(i)|-1}{k} (1-q)^k q^{n-|B(i)|-1-k} \Big] \times \frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} -$$

$$-\Big[\sum_{k=0}^{\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}-$$

$$-\Big[\sum_{k=0}^{\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

Or,

$$\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$+\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$-\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$-\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

Note that $\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)=-1$, $\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}>\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)\geq 0$ and $\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}<\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)\leq -2$.

Suppose first that $\Delta(i)\geq 0$. Then, the difference between the probabilities is

$$\frac{q^{\Delta(i)+1}-(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}\times$$

$$\times\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}[q^k(1-q)^{n-|B(i)|-1-k}-(1-q)^k q^{n-|B(i)|-1-k}]\Big]+$$

$$+\frac{1}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}\times$$

$$\times\Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}-1}\binom{n-|B(i)|-1}{k}[q^{k+\Delta(i)+1}(1-q)^{n-|B(i)|-1-k}+(1-q)^{k+\Delta(i)+1}q^{n-|B(i)|-1-k}]\Big]$$

The second addend is positive since $q\in(\frac{1}{2},1)$ (the multiplier and every element in the summation are positive) and $\Delta(i)\geq 0$ (there is at least one element in the summation). The multiplier in the first expression is positive since $q\in(\frac{1}{2},1)$ $(1>q>1-q>0)$ and $\Delta(i)\geq 0$ (the greater base matters). Finally, in the second part of the first expression, $k\geq\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}$. Since $\Delta(i)\geq 0$

we get $k > n - |B(i)| - 1 - k$. Also, by the summation bound $n - |B(i)| - 1 - k \geq 0$. Hence, $k > n - |B(i)| - 1 - k \geq 0$. $q \in (\frac{1}{2}, 1)$ means that $q^k(1-q)^{n-|B(i)|-1-k} > (1-q)^k q^{n-|B(i)|-1-k}$ and the whole expression is positive. Hence, if $s(i) = w$, $|B(i)| < n-1$ and $\Delta(i) \geq 0$ the probability that there are at least as many w signals as there are b signals is higher than the probability that there are at least as many b signals as there are w signals. Hence, if $s(i) = w$ and $\Delta(i) \geq 0$ then $a_2^i = W$ is optimal.

Suppose now that $\Delta(i) = -1$. Then, the difference between the probabilities is zero since the multiplier of the first addend is zero and the second addend does not exist. Hence, if $s(i) = w$, $|B(i)| < n-1$ and $\Delta(i) = -1$ the probability that there are at least as many w signals as there are b signals is equal to the probability that there are at least as many b signals as there are w signals. Hence, if $s(i) = w$ and $\Delta(i) \geq -1$ then $a_2^i \in \{W, B\}$ is optimal.

Suppose now that $\Delta(i) \leq -2$. Then, the difference between the probabilities is

$$\frac{q^{\Delta(i)+1} - (1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} \times$$

$$\times \Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1} \binom{n-|B(i)|-1}{k} [q^k(1-q)^{n-|B(i)|-1-k} - (1-q)^k q^{n-|B(i)|-1-k}] \Big] -$$

$$\frac{1}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} \times$$

$$\times \Big[\sum_{k=\frac{n}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{\frac{n}{2}-\frac{|B(i)|+\Delta(i)}{2}-2} \binom{n-|B(i)|-1}{k} [(1-q)^k q^{n+\Delta(i)-|B(i)|-k} + q^k(1-q)^{n+\Delta(i)-|B(i)|-k}] \Big]$$

The second part is positive since $q \in (\frac{1}{2}, 1)$ (the multiplier is positive as every element in the summation) and $\Delta(i) \leq -2$ (there is at least one element in the summation). The multiplier in the first expression is negative since $q \in (\frac{1}{2}, 1)$ $(1 > q > 1-q > 0)$ and $\Delta(i) \leq -2$. Finally, in the second part of the first expression, $k \geq \frac{n}{2} - \frac{|B(i)|+\Delta(i)}{2} - 1$. Since $\Delta(i) \leq -2$ we get $k > n - |B(i)| - 1 - k$. By the summation bounds we get that $k \leq n - |B(i)| - 1$. Hence, $k > n - |B(i)| - 1 - k \geq 0$. $q \in (\frac{1}{2}, 1)$ means that $q^k(1-q)^{n-|B(i)|-1-k} > (1-q)^k q^{n-|B(i)|-1-k}$. Thus, the second part of the first expression is positive and the whole expression is negative. Hence, if $s(i) = w$ and $\Delta(i) \leq -2$ the probability that there are at least as many w signals as there are b signals is lower than the probability that there are at least as many b signals as w signals. Thus, if $s(i) = w$, $|B(i)| < n-1$ and $\Delta(i) \leq -2$ a utility maximizer should choose $a_i^2 = B$.

A very similar proof shows that the same is true when $s(i) = w$ and n is odd. In this case there are no ties. The difference between the probability that there are more w signals than b signals and the probability that there are more b signals than w signals is

$$\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1} \binom{n-|B(i)|-1}{k} q^k(1-q)^{n-|B(i)|-1-k} \Big] \times \frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} +$$

$$+\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}-$$

$$-\Big[\sum_{k=0}^{\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-2}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}-$$

$$-\Big[\sum_{k=0}^{\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-2}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

Or,

$$\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$+\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$-\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}(1-q)^k q^{n-|B(i)|-1-k}\Big]\times\frac{q^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

$$-\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}q^k(1-q)^{n-|B(i)|-1-k}\Big]\times\frac{(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}$$

Note that $\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)=-1$, $\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}>\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)\geq 0$ and $\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}<\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1$ if and only if $\Delta(i)\leq -2$.

Suppose first that $\Delta(i)\geq 0$. Then, the difference between the probabilities is

$$\frac{q^{\Delta(i)+1}-(1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}\times$$

$$\times\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{n-|B(i)|-1}\binom{n-|B(i)|-1}{k}[q^k(1-q)^{n-|B(i)|-1-k}-(1-q)^k q^{n-|B(i)|-1-k}]\Big]+$$

$$\frac{1}{q^{\Delta(i)+1}+(1-q)^{\Delta(i)+1}}\times$$

$$\times\Big[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}-1}\binom{n-|B(i)|-1}{k}[q^{k+\Delta(i)+1}(1-q)^{n-|B(i)|-1-k}+(1-q)^{k+\Delta(i)+1}q^{n-|B(i)|-1-k}]\Big]$$

The second addend is positive since $q\in(\frac{1}{2},1)$ (the multiplier and every element in the summation are positive) and $\Delta(i) \geq 0$ (there is at least one element in the summation). The multiplier in the first expression is positive since $q \in (\frac{1}{2}, 1)$ and $\Delta(i) \geq 0$. Finally, in the second part of the first expression, $k \geq \frac{n+1}{2} - \frac{|B(i)|-\Delta(i)}{2}$. Since $\Delta(i) \geq 0$ we get $k > n - |B(i)| - 1 - k$. Also, by the summation bound $n - |B(i)| - 1 - k \geq 0$. Hence, $k > n - |B(i)| - 1 - k \geq 0$. $q \in (\frac{1}{2}, 1)$ means that $q^k(1-q)^{n-|B(i)|-1-k} > (1-q)^k q^{n-|B(i)|-1-k}$ and the whole expression is positive. Hence, if $s(i) = w$ and $\Delta(i) \geq 0$ the probability that there are more w signals than b signals is higher than the probability that there are more b signals than w signals. Hence, if $s(i) = w$ and $\Delta(i) \geq 0$ then $a_2^i = W$ is optimal.

Suppose now that $\Delta(i) = -1$. Then, the difference between the probabilities is zero since the multiplier of the first addend is zero and the second addend does not exist. Hence, if $s(i) = w$ and $\Delta(i) = -1$ the probability that there are more w signals than b signals is the same as the probability that there are more b signals than w signals. Hence, if $s(i) = w$ and $\Delta(i) = -1$ then $a_2^i \in \{W, B\}$ is optimal.

Suppose now that $\Delta(i) \leq -2$. Then, the difference between the probabilities is

$$\frac{q^{\Delta(i)+1} - (1-q)^{\Delta(i)+1}}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} \times$$

$$\times \Bigg[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-1}^{n-|B(i)|-1} \binom{n-|B(i)|-1}{k} [q^k(1-q)^{n-|B(i)|-1-k} - (1-q)^k q^{n-|B(i)|-1-k}] \Bigg] -$$

$$\frac{1}{q^{\Delta(i)+1} + (1-q)^{\Delta(i)+1}} \times$$

$$\times \Bigg[\sum_{k=\frac{n+1}{2}-\frac{|B(i)|-\Delta(i)}{2}}^{\frac{n+1}{2}-\frac{|B(i)|+\Delta(i)}{2}-2} \binom{n-|B(i)|-1}{k} [(1-q)^k q^{n+\Delta(i)-|B(i)|-k} + q^k(1-q)^{n+\Delta(i)-|B(i)|-k}] \Bigg]$$

The second part is positive since $q \in (\frac{1}{2}, 1)$ (the multiplier is positive as every element in the summation) and $\Delta(i) \leq -2$ (there is at least one element). The multiplier in the first expression is negative since $q \in (\frac{1}{2}, 1)$ and $\Delta(i) \leq -2$. Finally, in the second part of the first expression, $k \geq \frac{n+1}{2} - \frac{|B(i)|+\Delta(i)}{2} - 1$. Since $\Delta(i) \leq -2$ we get $k \geq \frac{n+1}{2} - \frac{|B(i)|}{2}$ and therefore $k > n - |B(i)| - 1 - k$. By the summation bounds we get that $k \leq n - |B(i)| - 1$. Hence, $k > n - |B(i)| - 1 - k \geq 0$. $q \in (\frac{1}{2}, 1)$ means that $q^k(1-q)^{n-|B(i)|-1-k} > (1-q)^k q^{n-|B(i)|-1-k}$. Thus, the second part of the first expression is positive and the whole expression is negative. Hence, if $s(i) = w$ and $\Delta(i) \leq -2$ the probability that there are more w signals than b signals is lower than the probability that there are more b signals than w signals. Thus, if $s(i) = w$ and $\Delta(i) \leq -2$ a utility maximizer should choose $a_i^2 = B$.

Thus, when $s(i) = w$, if $\Delta(i) \geq 0$ the optimal choice is $a_i^2 = W$, if $\Delta(i) \leq -2$ the optimal choice is $a_i^2 = B$ and if $\Delta(i) = -1$ both actions are optimal.

Suppose $s(i) = b$ and denote $\delta(i) = |B^1 B(i)| - |W^1 B(i)| = -\Delta(i)$. Thus, by the previous proof, when $s(i) = b$, if $\delta(i) \geq 0$ the optimal choice is $a_i^2 = B$, if $\delta(i) \leq -2$ then $a_i^2 = W$ and if $\delta(i) = -1$

then $a_i^2 \in \{B, W\}$. Hence, if $s(i) = b$ then $\Delta(i) \leq 0$ implies that the optimal choice is $a_i^2 = B$, $\Delta(i) \geq 2$ implies that the optimal choice is $a_i^2 = W$ and if $\Delta(i) = 1$ both actions are optimal. Finally, note that the optimal actions for the cases where there is no need to account for signals received by non-neighbors are compatible with the other cases.

Hence, we showed that for every $i \in N$:

1. Suppose $s(i) = w$. If $\Delta_1(i) \geq 0$ then $a_i^2 = W$, if $\Delta_1(i) = -1$ then $a_i^2 \in \{W, B\}$ and if $\Delta_1(i) \leq -2$ then $a_i^2 = B$.

2. Suppose $s(i) = b$. If $\Delta_1(i) \geq 2$ then $a_i^2 = W$, if $\Delta_1(i) = 1$ then $a_i^2 \in \{W, B\}$ and if $\Delta_1(i) \leq 0$ then $a_i^2 = B$.

□

Lemma 4

Lemma 4. *For every $i \in N$ and for every $j \in B(i)$:*

1. *If $h_i^3(j,1) = W$ and $h_i^3(j,2) = W$ then $\Delta_1(j) \geq -1$.*

2. *If $h_i^3(j,1) = W$ and $h_i^3(j,2) = B$ then $\Delta_1(j) \leq -1$.*

3. *If $h_i^3(j,1) = B$ and $h_i^3(j,2) = W$ then $\Delta_1(j) \geq 1$.*

4. *If $h_i^3(j,1) = B$ and $h_i^3(j,2) = B$ then $\Delta_1(j) \leq 1$.*

Proof. By Lemma 2, $h_i^3(j,1) = W$ implies $s(j) = w$. In addition, if $h_i^3(j,2) = W$ then $a_j^2 = W$. By Lemma 3 it must be that $\Delta_1(j) \geq -1$. Similarly for the other three cases. □

Lemma 5 (following Proposition 3.2 in Chandrasekhar et al. (2020))

Lemma 5. *Let $i \in N$. For every $t > 2$ and for every $j \in N$ such that $i \rhd j$: $P(\omega = WHITE | h_i^t) = P(\omega = WHITE | h_i^t(-j), h_i^t(j,1))$.*

Proof. The guess of agent j in period $t-1$, a_j^{t-1}, depends on h_j^{t-1} which includes the guesses of the agents in $B(j) \cup \{j\}$ in periods $\{1, \ldots, t-2\}$. Note that $t - 2 \geq 1$ and that, by Lemma 1 (and some abuse of notation), $h_j^{t-1}(j,1) = s(j)$. $i \rhd j$ implies $B(j) \cup \{j\} \subset B(i) \cup \{i\}$. Therefore, h_j^{t-1} is a restriction of h_i^{t-1} to the set $B(j) \cup \{j\}$. Since both agents are myopic Bayesian, agent i can use h_i^{t-1} to calculate a_j^{t-1} before he observes it (in the beginning of period t). Therefore, $h_i^t(j, t-1) = a_j^{t-1}$ does not reveal any new information to agent i. Since this holds for every $t > 2$, the only guess of agent j that agent i cannot calculate in advance is $a_j^1 = s(j)$. That is, $P(\omega = WHITE | h_i^t) = P(\omega = WHITE | h_i^t(-j), h_i^t(j,1))$. □

Definition 1

Definition 1 (Naïve Behavior). *For every $i \in N$:*

1. $t = 1$: *If $s(i) = w$ then $a_i^1 = W$ and if $s(i) = b$ then $a_i^1 = B$.*

2. $t > 1$:

   (a) *Suppose $a_i^{t-1} = W$. If $\Delta_{t-1}(i) \geq 0$ then $a_i^t = W$, if $\Delta_{t-1}(i) = -1$ then $a_i^t \in \{W, B\}$ and if $\Delta_{t-1}(i) \leq -2$ then $a_i^t = B$.*

   (b) *Suppose $a_i^{t-1} = B$. If $\Delta_{t-1}(i) \geq 2$ then $a_i^t = W$, if $\Delta_{t-1}(i) = 1$ then $a_i^t \in \{W, B\}$ and if $\Delta_{t-1}(i) \leq 0$ then $a_i^t = B$.*

Proposition 2

Proof. By Definition 1, each agent guesses by her private signal in the first period. Consider $i \in \hat{C}$. Agent i observes, at the end of period 1, more W guesses than B guesses, even if all her neighbores outside C guess B, since $b_i^{-C} < \gamma_C$. Therefore, she guesses $a_i^2 = W$. If for every member of $\hat{C}$ we have $b_i^{-C} < 2|\hat{C}| - m$ then for every member of $\hat{C}$ we have $(m - |\hat{C}|) + b_i^{-C} < |\hat{C}|$. That is, every member of $\hat{C}$ has more neighbores that are members of $\hat{C}$ then neighbors that are not in $\hat{C}$. Since every member of $\hat{C}$ guessed W in period 2, every member of $\hat{C}$ observes more Ws than Bs before guessing in period 3 and therefore guesses $a_i^3 = W$. This argument repeats itself in any subsequent period. Hence, $\forall i \in \hat{C}, \forall t \geq 2 : a_i^t = W$. □

C Predicted Dynamics: Formal Analysis

C.1 The Complete Network in Both Models

Result 1. *Suppose G is the complete network. By both models, $\forall i \in N$:*

1. $t = 1$*: If* $s(i) = w$ *then* $a_i^1 = W$ *and if* $s(i) = b$ *then* $a_i^1 = B$.
2. $\forall t > 1$*:*
   (a) *If* $|\{j \in N|s(j) = w\}| > |\{j \in N|s(j) = b\}|$*:* $a_i^t = W$.
   (b) *If* $|\{j \in N|s(j) = w\}| < |\{j \in N|s(j) = b\}|$*:* $a_i^t = B$.
   (c) *If* $|\{j \in N|s(j) = w\}| = |\{j \in N|s(j) = b\}|$*, there is no prediction. However, in the naïve model, if at any period* t*,* $|\{j \in N|a_j^t = W\}| > |\{j \in N|a_j^t = B\}|$ *then* $\forall s > t : a_i^s = W$ *and if at any period* t*,* $|\{j \in N|a_j^t = W\}| < |\{j \in N|a_j^t = B\}|$ *then* $\forall s > t : a_i^s = B$.

Proof. Let us begin with the Bayesian model. By Lemma 1, when $t = 1$, each player guesses by her signal. For $t > 1$, note that since G is complete, for every agent i, $B(i) = N\backslash\{i\}$. Suppose, first, that $|\{i \in N|s(i) = w\}| > |\{i \in N|s_i = b\}|$. If $s(i) = w$ then $\Delta_1(i) + 1 = |\{i \in N|s(i) = w\}| - |\{i \in N|s(i) = b\}|$. That is, $\Delta_1(i) + 1 > 0$ or $\Delta_1(i) > -1$. By Lemma 3, $a_i^2 = W$. If $s(i) = b$ then $\Delta_1(i) - 1 = |\{i \in N|s(i) = w\}| - |\{i \in N|s(i) = b\}|$. That is, $\Delta_1(i) - 1 > 0$ or $\Delta_1(i) > 1$. By Lemma 3, $a_i^2 = W$. Moreover, since there is no additional information to be revealed, if $a_i^2 = W$ then $a_i^t = W$ for all $t > 1$. Similarly, for the case where $|\{i \in N|s(i) = r\}| < |\{i \in N|s_i = b\}|$. When $|\{i \in N|s(i) = w\}| = |\{i \in N|s_i = b\}|$, by the same argument $a_i^2 = \{W, B\}$. Since there is no additional information to be revealed and since we assume nothing about behavior when there is a tie, we can predict nothing about the guesses for all $t > 1$.

By Definition 1 when $t = 1$, each player guesses by her signal. In the complete network, there is one all-inclusive clique. Hence, for every node i we have $b_i^{-C} = 0$. By Proposition 2, if $|\{i \in N|s(i) = w\}| > |\{i \in N|s(i) = b\}|$, $a_i^t = W$ for all $t \geq 2$. Similarly, for the case where $|\{i \in N|s(i) = w\}| < |\{i \in N|s(i) = b\}|$, $a_i^t = B$ for all $t \geq 2$. If $|\{i \in N|a_i^1 = W\}| = |\{i \in N|a_i^1 = B\}|$ then $a_i^2 = \{W, B\}$ and no prediction is available for subsequent periods. Note, however, that by similar considerations, once majority forms, it is sustain in all subsequent periods. □

C.2 Single Aggregator Networks in the Bayesian Model

Result 2. *Suppose G is a network with a single aggregator. By the Bayesian model:*

1. *For the aggregator:*
   (a) $t = 1$*: If* $s(i) = w$ *then* $a_i^1 = W$ *and if* $s(i) = b$ *then* $a_i^1 = B$.
   (b) $t > 1$*:*
      i. *If* $|\{j \in N|s(j) = w\}| > |\{j \in N|s(j) = b\}|$*,* $a_i^t = W$.
      ii. *If* $|\{j \in N|s(j) = w\}| < |\{j \in N|s(j) = b\}|$*,* $a_i^t = B$.

iii. *If* $|\{j \in N | s(j) = w\}| = |\{j \in N | s(j) = b\}|$, $a_I^t \in \{W, B\}$.

2. *For a non-aggregator* $j \in N \backslash \{i\}$*:*

   (a) $t = 1$*: If* $s(j) = w$ *then* $a_j^1 = W$ *and if* $s(j) = b$ *then* $a_j^1 = B$.

   (b) $t = 2$*:*

      i. Suppose $s(j) = w$. *If* $\Delta_1(j) \geq 0$ *then* $a_j^2 = W$*, if* $\Delta_1(j) = -1$ *then* $a_j^2 \in \{W, B\}$ *and if* $\Delta_1(j) \leq -2$ *then* $a_j^2 = B$.

      ii. Suppose $s(j) = b$. *If* $\Delta_1(j) \geq 2$ *then* $a_j^2 = W$*, if* $\Delta_1(j) = 1$ *then* $a_j^2 \in \{W, B\}$ *and if* $\Delta_1(j) \leq 0$ *then* $a_j^2 = B$.

   (c) $t > 2$*:* $a_j^t = a_i^{t-1}$.

Proof. By Lemma 1, all agents guess their private signal in $t = 1$. By Lemma 3, all agents guess the majority of their local neighborhood signals. By Lemma 5, the aggregator cares only about the signals of the other agents and therefore keeps his $t = 2$ guess until the game ends. Since the aggregator is strictly better informed than all the agents in the network, by definition, $\forall j \in N \backslash \{i\} : C(j) = \{i\}$. By Proposition 1, all the non-aggregators optimize by imitating the aggregator starting from $t = 3$. □

C.3 Star Networks in the Naïve Model

Claim 1. *Suppose* G *is a star network where* i *is the aggregator. For every period* t*, partition the leafs to* $L^t = \{k \in N \backslash \{i\} | a_k^t = a_i^t\}$ *and* $M^t = N \backslash (L^t \cup \{i\})$*. Then, by the naïve model,* $\forall k \in L^t : a_k^{t+1} = a_i^t$ *and* $\forall k \in M^t$ *there is no prediction for* a_k^{t+1}*. Let* $\neg a_i^t = B$ *if* $a_i^t = W$ *and* $\neg a_i^t = W$ *otherwise. Then,*

1. *If* $|L^t| + 1 > |M^t|$ *then* $a_i^{t+1} = a_i^t$.

2. *If* $|L^t| + 1 < |M^t|$ *then* $a_i^{t+1} = \neg a_i^t$.

3. *If* $|L^t| + 1 = |M^t|$ *then there is no prediction for* a_i^{t+1}.

Proof. L^t and M^t partition the leafs by their guesses in period t. By Definition 1, leafs in L^t observe the aggregator guess a_i^t and their own guess $a_k^t = a_i^t$ and therefore keep their guess ($a_k^{t+1} = a_i^t$). Leafs in M^t, on the other hand, observe the aggregator guess a_i^t and their own guess $a_k^t = \neg a_i^t$ and therefore there is no prediction for their guess in period $t + 1$. The aggregator observes all guesses made in period t. If she observes more guesses of a_i^t ($|L^t| + 1 > |M^t|$) she keeps her guess ($a_i^{t+1} = a_i^t$) while if she observes more guesses of $\neg a_i^t$ ($|L^t| + 1 < |M^t|$) she switches her guess ($a_i^{t+1} = \neg a_i^t$). If she observes a tie then we have no prediction for her next guess. □

Result 3. *Suppose* G *is a star network where* i *is the aggregator. By the naïve model:*

1. *If the aggregator recieved a private signal that corresponds to the majority's signal, then, all the participants that recieved the majority's signal guess by that signal in any future period. There is no prediction as to the guesses of the other participants. However, if at any period they guess by the majority's signal, they never switch back.*

2. *If the aggregator recieved a private signal that corresponds to the minority's signal:*

   (a) $t = 1$: *Every participant guesses by her own private signal.*

   (b) $t = 2$: *The aggregator switches. Participants that recieved the minority's signal keep their guesses and there is no prediction as to the guesses of those who recieved the majority's signal.*

   (c) $t \geq 3$:

      i. *If a_i^{t-1} coincides with the majority of period $t-1$'s guesses, the future dynamics is similar to case 1.*

      ii. *If a_i^{t-1} coincides with the minority of period $t-1$'s guesses, the dynamics is similar to case 2b.*

      iii. *If there is a tie in period $t-1$, the future dynamics is similar to case 3.*

3. *If there is a tie in the signal's distribution, participants that got the same signal as the aggregator keep their guesses. There is no prediction as to the guesses of the others (including the aggregator). If the aggregator has not switched then the dynamics continues as either 2(c)i or 2(c)iii. If he has switched every dynamics in 2c is possible.*

Proof. First, suppose the aggregator recieved a private signal that corresponds to the majority's signal. By Definition 1, L^1 includes all the leafs that got the majority's signal while M^1 includes all the leafs that got the minority's signal. Hence, by Claim 1 the aggregator and the members of L^1 keep their guess in period 2. In fact, $|L^2| + 1 \geq |L^1| + 1 > |M^1| \geq |M^2|$. Hence, by Claim 1 the members of L^1 and the aggregator, keep their guesses also in $t = 3$. This continues indefinitely. Note that once a participant becomes a member of L^t, she will always guess the same guess as the aggregator, that is, she will not switch to be a member of M^s for any $s > t$.

Second, suppose that the aggregator recieved a private signal that corresponds to the minority's signal. By Definition 1, every participant guesses by her private signal in $t = 1$. Also by Definition 1, the aggregator switches, participants that recieved the minority's signal keep their guesses and there is no prediction as to the guesses of those who recieved the majority's signal. Therefore, the relation between $|L^2|+1$ and $|M^2|$ is unclear. By Claim 1, if $|L^2|+1 > |M^2|$ (case 2(c)i) the aggregator and the members of L^2 keep their (correct) guesses and the members of M^2 may switch to that later in the game. Also by Claim 1, if $|L^2| + 1 < |M^2|$ (case 2(c)ii) the aggregator switches, the members of L^2 keep their guesses and the guesses of the members of M^2 cannot be predicted. Before the following period, the relation between $|L^3| + 1$ and $|M^3|$ is unclear. Again, by the same arguments, if the aggregator's guess coincides with the majority of period 3 guesses, the aggregator's switching is done and the majority of players keep their guess. Otherwise, the aggregator switches. This continues iteratively.

Finally, in any case of a tie (case 3 and case 2(c)iii), the considerations are almost identical to those used for the case where the aggregator recieved a private signal that corresponds to the minority's signal. The difference is that in the current period the aggregator's guess cannot be predicted (Claim 1). □

C.4 Connected Spokes Networks in the Naïve Model

Result 4. *Suppose G is a connected spokes network where i is the aggregator and $\{C_1, \ldots, C_m\}$ is the collection of cliques. Suppose agents behave according to the naïve model. In the first period all agents guess by their signal. For every clique $C_j \in \{C_1, \ldots, C_m\}$:*

1. $|\{k \in C_j | a_k^1 = W\}| > |\{k \in C_j | a_k^1 = B\}|$ *implies* $\forall k \in C_j \backslash \{i\}, \forall t \geq 2 : a_k^t = W$.
2. $|\{k \in C_j | a_k^1 = W\}| < |\{k \in C_j | a_k^1 = B\}|$ *implies* $\forall k \in C_j \backslash \{i\}, \forall t \geq 2 : a_k^t = B$.
3. $|\{k \in C_j | a_k^1 = W\}| = |\{k \in C_j | a_k^1 = B\}|$ *implies* $\forall k \in C_j \backslash \{i\} : a_k^2 = \{B, W\}$. *If in any subsequent period majority forms in C_j, all members, excluding i, follow the popular guess and never change.*
4. $\forall t \geq 2 :$
   - $|\{k \in N | a_k^{t-1} = W\}| > |\{k \in N | a_k^{t-1} = B\}|$ *implies* $a_i^t = W$.
   - $|\{k \in N | a_k^{t-1} = W\}| < |\{k \in N | a_k^{t-1} = B\}|$ *implies* $a_i^t = B$.
   - $|\{k \in N | a_k^{t-1} = W\}| = |\{k \in N | a_k^{t-1} = B\}|$ *implies* $a_i^t = \{B, W\}$.

Proof. By the definition of a connected spokes network, every non-aggregator is a member of a single clique in G. Moreover, all non-aggregators have no neighbores outside their clique. That is, $\forall j \in N \backslash \{i\} : b^{-C_j}(j) = 0$ where C_j is the clique that includes player j. Also note that, in every clique, the aggregator has at least $\frac{n}{2}$ outside the clique ($\min_{j \in \{1,\ldots,m\}} b^{C_j}(i) \geq \frac{n}{2}$).

Hence, for every $C \in \{C_1, \ldots, C_m\}$, unless $|C|$ is even and the signals are distributed equally among its members, $\hat{C} = N \backslash \{i\}$ since $\frac{n}{2} > \gamma_C > 0$. By Proposition 2, the non-aggregator members of the clique guess in the second period by the local majority in the first period and never change their guess afterwards.

If $|C|$ is even and the signals are distributed equally among its members then $\hat{C} = \emptyset$. In the second period the aggregator guesses by the global majority while the non-aggregators' guesses are undefined. If some majority is formed by the guesses of the clique members in the second period, the non-aggregators follow it and never change afterwards. If no majority forms, then again the aggregator guesses by the global majority while the non-aggregators' guesses are undefined, and so on, until a majority is formed.

The aggregator guess in each period by the majority across all players. Hence, if all non-aggregators cease to change their beliefs in period t, the aggregator ceases to change in period $t+1$, at the latest. If none of the cliques is such that $|C|$ is even and the signals are distributed equally among its members, then $t = 2$. □

C.5 One Gatekeeper Networks in the Naïve Model

Result 5. *Suppose G is a One Gatekeeper network. Denote the aggregator by i, the other core members by $C(G) = \{j_1, \ldots, j_m\}$ where m is even and the leafs by $K(G) = \{k_1, \ldots, k_n\}$ where*

$n = m+1$. For every period t, partition the leafs to $L^t = \{k \in K(G) | a_k^t = a_i^t\}$ and $M^t = K(G) \backslash L^t$. With no loss of generality, assume that w is the majority signal ($|\{h \in N | s(h) = w\}| > |\{h \in N | s(h) = b\}|$). By the naïve model:

1. *If $|\{h \in C(G) \cup \{i\} | s(h) = w\}| > |\{h \in C(G) \cup \{i\} | s(h) = b\}|$ and $s(i) = w$, then, from $t \geq 2$, all the participants guess correctly, excluding, maybe, the leafs that recieved incorrect signals. If at any period they guess correctly, they never switch back.*

2. *If $|\{h \in C(G) \cup \{i\} | s(h) = w\}| > |\{h \in C(G) \cup \{i\} | s(h) = b\}|$ and $s(i) = b$, then,*

    - *$t = 2$: The aggregator and the members of $C(G)$ guess correctly, leafs that recieved a wrong signal guess incorrectly and there is no prediction for the other leafs.*
    - *$t \geq 3$: Denote $D_L = M^1 \cap L^2$. If D_L is non-empty, the aggregator, the members of $C(G)$ and the members of D_L guess correctly. There is no prediction for the guesses of the other leafs. If D_L is empty, then the members of $C(G)$ guess correctly, but there is no prediction for the guesses of the aggregator and the leafs.*

3. *If $|\{h \in C(G) \cup \{i\} | s(h) = w\}| < |\{h \in C(G) \cup \{i\} | s(h) = b\}|$ and $s(i) = w$, then,*

    - *$t = 2$: The aggregator and at least $\frac{n+1}{2}$ leafs that recieved the correct signal guess correctly. The m members of $C(G)$ guess incorrectly. There is no prediction for the (at most $\frac{n-1}{2}$) leafs that recieved the incorrect signal (M^1).*
    - *$t \geq 3$: Suppose all M^1 members guessed correctly in $t = 2$. Then, the aggregator and all the leafs guess correctly while all the members in $C(G)$ guessed incorrectly. Nobody switches in any $t \geq 3$. If at least two M^1 members guessed incorrectly in $t = 2$. The aggregator switches. If any of the M^2 members keeps his second round guess, an incorrect majority forms where the aggregator, the members of $C(G)$ and L^3 will not switch back. Otherwise, there is a tie in $t = 2$ or $t = 3$ and there is no prediction for the aggregator's and the leafs' guesses.*

4. *If $|\{h \in C(G) \cup \{i\} | s(h) = w\}| < |\{h \in C(G) \cup \{i\} | s(h) = b\}|$ and $s(i) = b$, then,*

    - *$t = 2$: The m members of $C(G)$ and leafs that got the wrong signal guess incorrectly. The aggregator guesses correctly and there is no prediction for the guesses of at least $\frac{n+1}{2}$ leafs that recieved the correct signal.*
    - *$t \geq 3$: Suppose all the leafs guess correctly in $t = 2$ ($L^2 = K(G)$, $M^2 = \emptyset$). Then, the aggregator and all the leafs guess correctly while all the members in $C(G)$ guessed incorrectly. Nobody switches in any $t \geq 3$. Now suppose that M^2 includes at least two agents. There is an incorrect majority in $t = 2$. Since the aggregator switches to the wrong guess, if any member of M^2 keep his guess, there is a wrong majority and nobody switches in any $t \geq 3$. Otherwise, there is a tie and no prediction for the aggregator's and the leafs' guesses.*

Proof. First, suppose that the signal recieved by the majority of all agents is the same as the signal recieved by the majority of the members in $C(G) \cup \{i\}$. By Proposition 2, all members of $C(G)$ follow the majority signal from $t \geq 2$. If the aggregator recieved the majority signal, then, since there are $m + n + 1 = 2(m + 1)$ nodes, there is at least one leaf that also recieved the majority signal. In $t = 2$ the aggregator keeps her guess (since it is the majority signal) and also every leaf ($k \in K(G)$) that recieved the majority signal keeps his guess since $a_k^1 = a_i^1$. Therefore, the aggregator guesses with the majority of the guesses and does not switch at any subsequent period. The leafs that recieved the majority signal do not switch. Hence, if the aggregator recieved the majority signal, from $t \geq 2$, all agents, excluding the leafs that recieved the minority signal, guess correctly. There is no prediction for the leafs that recieved the minority signal, but once they switch they do not switch back.

If the aggregator recieved the minority signal then, since there are $m + n + 1 = 2(m + 1)$ nodes, there are at least two leafs that recieved the majority signal. In $t = 2$, by Proposition 2, all members of $C(G)$ follow the majority signal. The aggregator also follows the majority signal. Every leaf ($k \in K(G)$) that recieved the minority signal keeps his guesses since $a_k^1 = a_i^1$. There is no prediction regarding the leafs that recieved the majority signal. If at least one of them guesses by the majority signal, then there is a majority for this guess which remains in all subsequent periods (due to Definition 1 and Proposition 2). However, if all the leafs that recieved the majority signal switch to the minority signal there is a tie in $t = 2$, where the aggregator and the core members guess by the majority signal and the leafs guess by the minority signal. In this case there is no prediction for the subsequent guesses of the aggregator and the leafs.

Next, suppose that the signal recieved by the majority of all agents is not the signal recieved by the majority of the members in $C(G) \cup \{i\}$. By Proposition 2, all members of $C(G)$ follow the minority signal from $t \geq 2$. Suppose the aggregator recieved the majority signal. If all of M^1 leafs switch in $t = 2$, then there is a correct majority since the aggregator and all the leafs guess correctly. In this case, no agents switch at any subsequent period. However, if at least two of the leafs that recieved the wrong signal, guess incorrectly also in $t = 2$, then the wrong majority forms in the second period. As a result, in the third period, the aggregator switches. If any one of the leafs guesses incorrectly in $t = 3$, an incorrect majority forms. This incorrect majority will not be resolved. Other cases lead to ties (e.g. when one M^1 leaf guess incorrectly in $t = 2$ or when no leaf guesses incorrectly while the aggregator switched in $t = 3$). In these cases the dynamics cannot be predicted.

Finally, suppose the aggregator recieved the minority signal. Therefore, he switches in $t = 2$. If $L^2 = K(G)$ and $M^2 = \emptyset$ there is a correct majority where no agent wants to switch in $t \geq 2$. Otherwise, in $t = 2$, the aggregator guesses correctly, but the members of M^2 guess incorrectly as do the m agents in $C(G)$. Therefore, if M^2 includes at least two agents, there is an incorrect majority in $t = 2$ and the aggregator switches in $t = 3$. For this switch to become permanent, at least one member of M^2 should guess incorrectly in $t = 3$. Otherwise, there is a tie and there is no prediction regarding the future guesses of the aggregator and the leafs. □

C.6 Symmetric Core Periphery Networks in Both Models

Let $N = N_1 \cup N_2$ where $|N_1| = \frac{n}{2}$ and $|N_2| = \frac{n}{2}$. A Symmetric Core Periphery network is defined by a function $F : N_1 \to N_2$ which is a bijective function that assigns a distinct member of N_2 to each member of N_1. Denote its inverse by $G : N_2 \to N_1$. Then, $\forall i \in N_1 : B(i) = [N_1 \backslash \{i\}] \cup \{F(i)\}$ and $\forall j \in N_2 : B(j) = \{G(j)\}$. Denote $\hat{\Delta} = |\{j \in N_1 | s(j) = w\}| - |\{j \in N_1 | s(j) = b\}|$ and $\tilde{\Delta} = |\{i \in N | s(i) = w\}| - |\{i \in N | s(i) = b\}|$.

Result 6. *Suppose G is a Core Periphery network. By both models:*

1. *If $\hat{\Delta} > 1$, $\forall i \in N_1, \forall t \geq 2 : a_i^t = W$.*
2. *If $\hat{\Delta} < -1$, $\forall i \in N_1, \forall t \geq 2 : a_i^t = B$.*
3. *If $\hat{\Delta} = 0$, then*
   (a) *$\forall i \in N_1 : a_i^2 = a_{F(i)}^1$.*
   (b) *If $\tilde{\Delta} > 0$, $\forall i \in N_1, \forall t \geq 3 : a_i^t = W$.*
   (c) *If $\tilde{\Delta} < 0$, $\forall i \in N_1, \forall t \geq 3 : a_i^t = B$.*
   (d) *If $\tilde{\Delta} = 0$ there is no prediction from $t \geq 3$.*
4. *If $\hat{\Delta} \in \{-1, 1\}$, there is no prediction from $t \geq 2$.*
5. *$\forall : j \in N_2$:*
   (a) *In the Bayesian model: $\forall t > 2 : a_j^t = a_{G(j)}^{t-1}$.*
   (b) *In the naïve model: there is no prediction from $t \geq 2$ unless (i) If $s(j) = w$, $\hat{\Delta} > 1$ and $s(G(j)) = w$ then $\forall t \geq 2 : a_j^t = W$ or (ii) If $s(j) = b$, $\hat{\Delta} < -1$ and $s(G(j)) = b$ then $\forall t \geq 2 : a_j^t = B$.*

Proof. We begin with the Bayesian model. First, consider the leafs (members of N_2). By Lemma 1 every agent in the network guesses her signal in the first round. By Lemma 3, in the second round they guess their signal if $a_j^1 = a_{G(j)}^1$ and we have no prediction for their guess in the case where $a_j^1 \neq a_{G(j)}^1$. By Proposition 1, $\forall j \in N_2, \forall t > 2 : a_j^t = a_{G(j)}^{t-1}$.

Next, we consider the core members (members of N_1). By Lemma 1 every agent in the network guesses her signal in the first round. Each member of the core observes, before the second round, the guesses of her $\frac{n}{2}$ neighbors: $\frac{n}{2} - 1$ are the other core members and the additional neighbor is the periphery member that is linked to her. Suppose $\hat{\Delta} > 1$. Consider some core member $i \in N_1$. If $s(i) = w$ and $s(F(i)) = b$ then $\Delta_1(i) = \hat{\Delta} - 1 - 1 \geq 0$. In addition, if $s(i) = w$ and $s(F(i)) = w$ then $\Delta_1(i) = \hat{\Delta} - 1 + 1 \geq 0$. If on the other hand $s(i) = b$ then if $s(G(i)) = b$ we get $\Delta_1(i) = \hat{\Delta} + 1 - 1 \geq 2$ while if $s(G(i)) = w$ we get $\Delta_1(i) = \hat{\Delta} + 1 + 1 \geq 2$. Hence, if $s(i) = w$ we get $\Delta_1(i) \geq 0$ while if $s(i) = b$ we get $\Delta_1(i) \geq 2$. Therefore, by Lemma 3, the optimal guess for every core member $i \in N_1$ in the second round when $\hat{\Delta} > 1$ is $a_i^2 = W$. Importantly, note that when $\hat{\Delta} > 1$, the optimal guess of each core member $i \in N_1$ is independent of the signal of her non-core neighbor. This implies that in the case of $\hat{\Delta} > 1$, the core members know each others' second round guesses before they observe it. In addition, by Lemma 5, they learn nothing from the guess of their non-core neighbor in the second round. As a result, in the case of $\hat{\Delta} > 1$, the core members learn no new information at the end of round 2. Since they are myopic (so there is no manipulation to extract additional information in the future) and since they were not indifferent in their second round guess, their optimal guess in the third round is $a_i^3 = W$. The same argument determines the optimal behavior in subsequent periods as well. Therefore, if $\hat{\Delta} > 1$, $\forall i \in N_1, \forall t \geq 2 : a_i^t = W$. Similarly, if $\hat{\Delta} < -1$, $\forall i \in N_1, \forall t \geq 2 : a_i^t = B$.

Suppose that $\frac{n}{2}$ is even. Then, the only case left to study is when $\hat{\Delta} = 0$. In this case, by Lemma 3, each core member $i \in N_1$ guesses in the second round by the signal of her leaf, $a_i^2 = a_{F(i)}^1$, since the signals of the core members cancel out. Therefore, before the guesses of the third round, every member of the core knows the exact distribution of signals in the network. Hence, they all agree and guess the overall majority (if such exists) beginning at period 3. If there is a tie, we cannot predict their behavior.

Finally, suppose that $\frac{n}{2}$ is odd. Then, if $1 \geq \hat{\Delta} \geq -1$ it must be that $\hat{\Delta} \in \{-1, 1\}$. Consider agent $i \in N_1$, denote the majority signal among the members of the core by $m \in \{w, b\}$. By Lemma 2, agent i knows $s(F(i))$. If $s(F(i)) = m$ then $a_i^2 = M$, that is, the guess reflects the majority signal. However, if $s(F(i)) \neq m$ then agent i observes a tie in her a local neighborhood and therefore $a_i^2 \in \{W, B\}$. That is, every core member who guessed not M has a periphery agent connected to it that got a signal which is not m. However, if a core member guessed M, the signal received by the periphery agent connected to her cannot be deduced. Since we assume nothing on the choice of the agents in case of a tie, we cannot predict the behavior of the core agents.

Next, we study the Naïve model. Definition 1 implies that each agent guesses by her own private signal. Note that for every member of the core ($i \in N_1$) we have $b^{-C}(i) = 1$ since each core member maintains a link with one periphery member. Therefore, if $\gamma_C = \hat{\Delta} > 1$ then, by Proposition 2, $\forall i \in N_1, \forall t \geq 2 : a_i^t = W$. The leafs guess in the first round by their private signal. In the second round they stick to the same guess if they recieved the same private signal as their core-member neighbor. Otherwise, by Definition 1, we cannot predict their guess. For $t \geq 3$, a leaf that guessed W in the second round will guess W throughout the game. Otherwise we cannot predict their guesses.

Similarly, if $\hat{\Delta} < -1$, $\forall i \in N_1, \forall t \geq 2 : a_i^t = B$. The leafs behavior is such that if both a leaf and his core-member neighbor recieved a private signal b, he guesses B throughout the game, otherwise his behavior cannot be predicted.

Suppose that $\frac{n}{2}$ is even. Then, the only case left to study is where $\hat{\Delta} = 0$. In this case, by Definition 1, each core member $i \in N_1$ guesses in the second round by the signal of his leaf, $a_i^2 = a_{F(i)}^1$, since the signals of the core members cancel out. Since $\frac{n}{2}$ is even, if there is no global tie, it must be that $\tilde{\Delta} \geq 2$ or $\tilde{\Delta} \leq -2$. In addition, since the signals of the core members cancel out, then the difference is attributed to the leafs' signals. Furthermore, since we have shown that in this case, the private signals of the leafs are the guesses of the respective cores in the second round, if there is no global tie, if $\tilde{\Delta} \geq 2$ all core members guess W throughout the game while if $\tilde{\Delta} \leq -2$ all core members guess B throughout the game, independently of the second round guess of their respective periphery neighbores. Otherwise, if there is global tie ($\tilde{\Delta} = 0$) then in the third round, the core members guess as their respective leaf neighbors guessed in the second round ($a_i^3 = a_{F(i)}^2$). Since we cannot predict the guesses of the leafs in the second round, we cannot predict the guesses of the core members from the third round onward.

Finally, suppose that $\frac{n}{2}$ is odd. The cases left to characterize are where $\hat{\Delta} \in \{-1, 1\}$. Since in many cases we cannot predict the second round guess of the leafs we cannot determine the third round guesses of the core members in this case. Note, in relation to the cases where we cannot determine the behavior of the core member, that if at any round we get $\hat{\Delta} \geq 2$ or $\hat{\Delta} \leq -2$ the guesses of all core members are set to the popular guess throughout the game. □

C.7 Two Cores with One Link in the Bayesian Model

Let $N = N_1 \cup N_2$ where $|N_1| = \frac{n}{2}$ and $|N_2| = \frac{n}{2}$. N_1 and N_2 are cliques of size $\frac{n}{2}$. Let agents $i \in N_1$ and agent $j \in N_2$ be the two connectors, that is, $E \cap \{kl | k \in N_1, l \in N_2\} = \{ij\}$. Denote $N_1^{-i} = N_1 \backslash \{i\}$, $N_1^{+j} = N_1 \cup \{j\}$, $N_2^{-j} = N_2 \backslash \{j\}$ and $N_2^{+i} = N_2 \cup \{i\}$. Denote the number of w signals in N_1, $|\{l \in N_1 | s(l) = w\}|$, by n_{1w}. Similarly, we denote the number of w signals in N_1^{-i}, N_1^{+j}, N_2, N_2^{-j} and N_2^{+i} by n_{1w}^{-i}, n_{1w}^{+j}, n_{2w}, n_{2w}^{-j} and n_{2w}^{+i}, respectively. Finally, denote the number of w signals among the connectors by $w_{ij} = |\{k \in \{i, j\} | s(k) = w\}|$ (note that w_{ij} is known to both agent i and agent j).

Result 7. *G is a Two Cores One Link network with $n = 18$ agents and the probability to receive a correct signal is $q = 0.7$. By the Bayesian model:*

1. *$\forall k \in N$: If $s(k) = w$ then $a_k^1 = W$, otherwise, $a_k^1 = B$.*
2. *$\forall k \in N_1^{-i}$: If $n_{1w} > 4$ then $a_k^2 = W$, otherwise, $a_k^2 = B$. In addition, $\forall t > 2$: $a_k^t = a_i^{t-1}$.*
3. *$\forall k \in N_2^{-j}$: If $n_{2w} > 4$ then $a_k^2 = W$, otherwise, $a_k^2 = B$. In addition, $\forall t > 2$: $a_k^t = a_j^{t-1}$.*
4. *Second round for the connectors:*
    - *If $n_{1w}^{+j} > 5$ then $a_i^2 = W$, if $n_{1w}^{+j} < 5$ then $a_i^2 = B$, otherwise, $a_i^2 \in \{B, W\}$.*
    - *If $n_{2w}^{+i} > 5$ then $a_j^2 = W$, if $n_{2w}^{+i} < 5$ then $a_j^2 = B$, otherwise, $a_j^2 \in \{B, W\}$.*
5. *If $a_i^2 = a_j^2$ then $\forall t > 2$: $a_i^t = a_j^t = a_i^2$.*[6]
6. *If $a_i^1 \neq a_j^1$ and $a_i^2 \neq a_j^2$ then with no loss of generality assume $a_i^2 = W$. Denote $K = n_{1w}^{-i}$:*

[6] A failure may occur only if three subjects in N_1^{-i}, three subjects in N_2^{-j} and the two connectors recieve the same signal while all the others recieve the opposite signal and $a_i^2 = a_j^2 = a_i^1 = a_j^1$. The probability for this distribution of signals is 0.69%. Since we assume no tie breaking rule, 0.69% is an upper bound for the unconditional probability of such a failure.

(a) *If* $n_{2w}^{-j} > 8 - K$ *then denote* $\bar{K} = 7 - n_{2w}^{-j}$*:*

- *If* $\bar{K} \geq 4$*:* $\forall(\bar{K}-1) \geq t \geq 3 : a_i^t = W, a_j^t = B$.
- $\forall t \geq \bar{K} : a_i^t = a_j^t = W$.

(b) *If* $n_{2w}^{-j} < 8 - K$ *then*

- *If* $K \geq 5$*:* $\forall(K-2) \geq t \geq 3 : a_i^t = W, a_j^t = B$.
- $\forall t \geq K - 1 : a_i^t = a_j^t = B$.

(c) *If* $n_{2w}^{-j} = 8 - K$ *then*

- *If* $K = 4$ *then* $a_i^3 = B$*,* $a_j^3 = W$ *and* $\forall t \geq 4 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$.
- *If* $K \in \{5, 6, 7\}$ *then* $\forall(K-2) \geq t \geq 3 : a_i^t = W, a_j^t = B$ *and* $a_i^{K-1} = B, a_j^{K-1} = W$ *and* $\forall t \geq K : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$.
- *If* $K = 8$ *then* $\forall(K-2) \geq t \geq 3 : a_i^t = W, a_j^t = B$ *and* $\forall t \geq 7 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$.

7. *If* $a_i^1 = a_j^1$ *and* $a_i^2 \neq a_j^2$ *then with no loss of generality assume* $a_i^1 = a_j^1 = a_i^2 = W$*. Denote* $K = n_{1w}^{-i}$*:*

(a) *If* $n_{2w}^{-j} > 7 - K$ *then denote* $\bar{K} = 6 - n_{2w}^{-j}$*:*

- *If* $\bar{K} \geq 4$*:* $\forall(\bar{K}-1) \geq t \geq 3 : a_i^t = W, a_j^t = B$.
- $\forall t \geq \bar{K} : a_i^t = a_j^t = W$.

(b) *If* $n_{2w}^{-j} < 7 - K$ *then*

- *If* $K = 3$ *and* $n_{2w}^{-j} < 3$*:* $\forall t \geq 3 : a_i^t = a_j^t = B$.
- *If* $K = 3$ *and* $n_{2w}^{-j} = 3$*:* $a_i^3 = B$*,* $a_j^3 = W$ *and* $\forall t \geq 4 : a_i^t = a_j^t = B$.
- *If* $K \geq 5$*:* $\forall(K-2) \geq t \geq 3 : a_i^t = W, a_j^t = B$.
- *If* $K \geq 4$*:* $\forall t \geq K - 1 : a_i^t = a_j^t = B$.

(c) *If* $n_{2w}^{-j} = 7 - K$ *then*

- *If* $K = 4$ *then*
  - $a_i^3 = B$ *and* $a_j^3 = W$.
  - $\forall t \geq 4 : a_i^t \in \{B, W\}$*. Denote* $t_i^W = \min\{t \geq 4 | a_i^t = W\}$.
  - $\forall t_i^W \geq t \geq 4 : a_j^t = B$ *and* $\forall t > t_i^W : a_j^t \in \{B, W\}$
- *If* $K \in \{5, 6\}$ *then* $\forall(K-2) \geq t \geq 3 : a_i^t = W, a_j^t = B$ *and* $a_i^{K-1} = B, a_j^{K-1} = W$ *and* $\forall t \geq K : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$.
- *If* $K = 7$ *then*
  - $a_i^3 = a_i^4 = a_i^5 = W$ *and* $a_j^3 = a_j^4 = a_j^5 = B$.
  - $\forall t \geq 6 : a_i^t \in \{B, W\}$*. Denote* $t_i^B = \min\{t \geq 6 | a_i^t = B\}$.
  - $\forall t_i^B \geq t \geq 6 : a_j^t = W$ *and* $\forall t > t_i^B : a_j^t \in \{B, W\}$

Proof. All agents are myopic Bayesian utility maximizers and that is a common knowledge. By Lemma 1 all agents are guessing their signal in the first period (Result 7.1) and by Lemma 3 in the second period each agent chooses her action by the most popular signal in her local neighborhood (Result 7.4 and the first part of results 7.2 and 7.3). By Proposition 1 the non-connectors imitate the connector to whom they are linked (the second part of results 7.2 and 7.3). Therefore, the dynamics is determined by the actions of agent i and agent j, the connectors, starting from $t = 3$.

First, suppose the connectors agree in the second round, that is, $a_i^2 = a_j^2$. With no loss of generality, consider the case where $a_i^2 = a_j^2 = W$. At the beginning of the third round, agent i knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 5 - w_{ij}$. Recall that agent i herself guessed W in the second round, therefore, $n_{1w}^{+j} \geq 5$. Thus, the only case where agent i may attribute positive probability to the event that the total number of w signals is lower than 9 (the minimum required for $a_i^3 = a_i^2 = W$ to be optimal) is when $n_{1w}^{-i} = 3$ and $w_{ij} = 2$. In this case, if $n_{2w}^{-j} = 3$ then the unique optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 5$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 3, w_{ij} = 2, n_{2w}^{-j} \geq 3) \approx 0.21^{7} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 3, w_{ij} = 2, n_{2w}^{-j} \geq 3) \approx 0.6^{8}$$

Therefore, in the case where $a_i^2 = a_j^2 = W$, the optimal guess for agent i is $a_i^3 = W$. This is naturally true also for the other connector, that is, $a_j^3 = W$. Note that (i) The non-connectors are non-informative to the connectors starting from the second round (Lemma 5) and (ii) Each connector knows already after the second round that the other connector observes at least $5 - w_{ij}$ supporting signals that she cannot observe. Hence, the third round provides no new information. Therefore, both will keep their guesses unchanged until the end of the game (Result 7.5). The unconditional probability for the signal distribution to be such that $n_{1w}^{-i} = 3$, $n_{2w}^{-j} = 3$ and $w_{ij} = 2$ is approximately 0.34%. By symmetry, the probability for a similar distribution for the b signals is identical. Therefore, the probability for this case is approximately 0.69%. Since no prior on the tie breaking rule is assumed, 0.69% serves as an upper bound for the probability of failure.

Next, we study the case where the connectors disagree both in the first round ($a_i^1 \neq a_j^1$, that is, $w_{ij} = 1$) and in the second round ($a_i^2 \neq a_j^2$). With no loss of generality, let us consider the case where $a_i^2 = W$ and focus on the considerations of agent i. At the beginning of the third round, agent i knows that since agent j guessed B in the second round it must be that $n_{2w}^{-j} \leq 4$.

Let us first attend to the case where $n_{1w}^{-i} = 5$. In this case, $a_i^3 = W$ is the unique optimal guess

[7]
$$\frac{\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{3} 0.7^3 0.3^5 + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{3} 0.7^5 0.3^3}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[8]
$$\frac{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

for agent i if $n_{2w}^{-j} = 4$ while $a_i^3 = B$ is the unique optimal guess for agent i if $n_{2w}^{-j} \leq 2$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 4 | n_{1w}^{-i} = 5, w_{ij} = 1, n_{2w}^{-j} \leq 4) \approx 0.44^{9} \qquad p(n_{2w}^{-j} \leq 2 | n_{1w}^{-i} = 5, w_{ij} = 1, n_{2w}^{-j} \leq 4) \approx 0.31^{10}$$

Therefore, in the case where $a_i^1 \neq a_j^1$, $a_i^2 \neq a_j^2$ and $a_i^2 = W$ if $n_{1w}^{-i} = 5$ the optimal guess for agent i is $a_i^3 = W$. Since the draws are independent, the probability that $a_i^3 = W$ is optimal, monotonically increases with the number of agents in N_1^{-i} that recieve the signal w. Therefore, if $n_{1w}^{-i} \geq 5$ the optimal guess for agent i is $a_i^3 = W$. If $n_{1w}^{-i} \leq 3$, agent i would have agreed with agent j on B in the second round. Therefore, to complete the optimality analysis of the third round when $a_i^1 \neq a_j^1$, $a_i^2 \neq a_j^2$ and $a_i^2 = W$, it is left to consider the case where $n_{1w}^{-i} = 4$. However, in this case, $a_i^3 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^3 = B$. That is, agent i switches to the second round guess of agent j if and only if $n_{1w}^{-i} = 4$.

If both connectors switch, then it must be that there are exactly 9 signals of each color. That is, $\forall t \geq 4 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. In addition, if only one of the connectors switches, it is clear that the switch was in the correct direction. That is, if $a_i^2 \neq a_i^3 = a_j^3 = a_j^2$ then $\forall t \geq 4 : a_i^t = a_j^t = a_j^2$ and if $a_i^2 = a_i^3 = a_j^3 \neq a_j^2$ then $\forall t \geq 4 : a_i^t = a_j^t = a_i^2$. As a result, it is left to study the guesses of the connectors from the fourth round onward when both did not switch between the second round and the third round, $a_i^1 \neq a_j^1$, $a_i^2 \neq a_j^2$ and $a_i^3 \neq a_j^3$.

Let us focus on the considerations of agent i in the fourth round. At the beginning of the fourth round, agent i knows that agent j guessed B in the second and third round only if $n_{2w}^{-j} \leq 3$. Let us first attend to the case where $n_{1w}^{-i} = 6$. In this case, $a_i^4 = W$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 3$ while $a_i^4 = B$ is the unique optimal guess for agent i if $n_{2w}^{-j} \leq 1$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 6, w_{ij} = 1, n_{2w}^{-j} \leq 3) \approx 0.65^{11} \qquad p(n_{2w}^{-j} \leq 1 | n_{1w}^{-i} = 6, w_{ij} = 1, n_{2w}^{-j} \leq 3) \approx 0.12^{12}$$

9

$$\frac{\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{4} 0.7^4 0.3^4 + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{4} 0.7^4 0.3^4}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

10

$$\frac{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

11

$$\frac{\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{3} 0.7^3 0.3^5 + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{3} 0.7^5 0.3^3}{\sum_{k=0}^{3} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

12

$$\frac{\sum_{k=0}^{1} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=0}^{3} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

Therefore, in the case where $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 \neq a_j^3 = a_j^2$ and $a_i^2 = W$ if $n_{1w}^{-i} = 6$ the optimal guess for agent i is $a_i^4 = W$. Using a similar argument as before, if $n_{1w}^{-i} \geq 6$ the optimal guess for agent i is $a_i^4 = W$. If $n_{1w}^{-i} \leq 4$, agent i would have agreed with agent j on B in the second or third round (or implement her tie breaking choice). Therefore, to complete the optimality analysis of the fourth round when $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 \neq a_j^3 = a_j^2$ and $a_i^2 = W$, it is left to consider the case where $n_{1w}^{-i} = 5$. However, in this case, $a_i^4 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^4 = B$. That is, agent i switches to the second round guess of agent j if and only if $n_{1w}^{-i} = 5$.

If both connectors switch, then it must be that there are exactly 9 signals of each color. That is, $\forall t \geq 5 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. In addition, if only one of the connectors switches, it is clear that the switch was in the correct direction. That is, if $a_i^2 = a_i^3 \neq a_i^4 = a_j^4 = a_j^3 = a_j^2$ then $\forall t \geq 4 : a_i^t = a_j^t = a_j^2$ and if $a_i^2 = a_i^3 = a_i^4 = a_j^4 \neq a_j^3 = a_j^2$ then $\forall t \geq 4 : a_i^t = a_j^t = a_i^2$. As a result, it is left to study the guesses of the connectors from the fifth round onward when both did not switch between the third round and the fourth round, $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 = a_i^4 \neq a_j^4 = a_j^3 = a_j^2$.

Let us focus on the considerations of agent i in the fifth round. At the beginning of the fifth round, agent i knows that agent j guessed B in the second, third and fourth round only if $n_{2w}^{-j} \leq 2$. Let us first attend to the case where $n_{1w}^{-i} = 7$. In this case, $a_i^5 = W$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 2$ while $a_i^5 = B$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 0$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} = 2 | n_{1w}^{-i} = 7, w_{ij} = 1, n_{2w}^{-j} \leq 2) \approx 0.8^{13} \qquad p(n_{2w}^{-j} \leq 1 | n_{1w}^{-i} = 7, w_{ij} = 1, n_{2w}^{-j} \leq 2) \approx 0.03^{14}$$

Therefore, in the case where $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 = a_i^4 \neq a_j^4 = a_j^3 = a_j^2$ and $a_i^2 = W$ if $n_{1w}^{-i} = 7$ the optimal guess for agent i is $a_i^5 = W$. Using a similar argument as before, if $n_{1w}^{-i} \geq 7$ the optimal guess for agent i is $a_i^5 = W$. If $n_{1w}^{-i} \leq 5$, agent i would have agreed with agent j on B in the second, third or fourth round (or implement her tie breaking choice). Therefore, to complete the optimality analysis of the fifth round when $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 = a_i^4 \neq a_j^4 = a_j^3 = a_j^2$ and $a_i^2 = W$, it is left to consider the case where $n_{1w}^{-i} = 6$. However, in this case, $a_i^5 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^5 = B$. That is, agent i switches to the second round guess of agent j if and only if $n_{1w}^{-i} = 6$.

If both connectors switch, then it must be that there are exactly 9 signals of each color. That is, $\forall t \geq 6 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. In addition, if only one of the connectors switches, it is

[13]

$$\frac{\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{2} 0.7^2 0.3^6 + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{2} 0.7^6 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

[14]

$$\frac{\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{0} 0.7^0 0.3^8 + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{0} 0.7^8 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{2}{1} 0.7^1 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

clear that the switch was in the correct direction. As a result, it is left to study the guesses of the connectors from the sixth round onward when both did not switch between the fourth round and the fifth round, $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 = a_i^4 = a_i^5 \neq a_j^5 = a_j^4 = a_j^3 = a_j^2$.

Let us focus on the considerations of agent i in the sixth round. At the beginning of the sixth round, agent i knows that agent j guessed B in the second, third, fourth and fifth round only if $n_{2w}^{-j} \leq 1$. Note that if $n_{1w}^{-i} \leq 6$, agent i would have agreed with agent j on B in the second, third, fourth or fifth round (or implement her tie breaking choice). If $n_{1w}^{-i} = 7$ then $a_i^6 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^6 = B$. However, if $n_{1w}^{-i} = 8$, $a_i^6 = B$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^6 = W$. If both connectors switch, then it must be that there are exactly 9 signals of each color. That is, $\forall t \geq 7 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. In addition, if only one of the connectors switches, it is clear that the switch was in the correct direction. As a result, it is left to study the guesses of the connectors from the seventh round onward when both did not switch between the fifth round and the sixth round, $a_i^1 \neq a_j^1$, $a_i^2 = a_i^3 = a_i^4 = a_i^5 = a_i^6 \neq a_j^6 = a_j^5 = a_j^4 = a_j^3 = a_j^2$. In that case, at the beginning of the seventh round, both agents understand that $n_{1w}^{-i} = 8$ and $n_{2w}^{-j} = 0$ since otherwise switches would happen earlier. Hence, it must be that there are exactly 9 signals of each color. That is, $\forall t \geq 7 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. This completes the proof of result 7.6.

Finally, we attend to the case where the connectors agree in the first round ($a_i^1 = a_j^1$, that is, $w_{ij} \in \{0, 2\}$) but not in the second round ($a_i^2 \neq a_j^2$). With no loss of generality, let us consider the case where $w_{ij} = 2$ and $a_i^2 = W$ and study the considerations of both agents in the third round. At the beginning of the third round, agent i knows that since agent j guessed B in the second round it must be that $n_{2w}^{-j} \leq 3$. At the same time, agent j knows that since agent i guessed W in the second round it must be that $n_{1w}^{-i} \geq 3$.

Let us first attend to the choice of agent i in the third round. We begin with the case where $n_{1w}^{-i} = 5$. In this case, $a_i^3 = W$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 3$ while $a_i^3 = B$ is the unique optimal guess for agent i if $n_{2w}^{-j} \leq 1$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 5, w_{ij} = 2, n_{2w}^{-j} \leq 3) \approx 0.65^{15} \qquad p(n_{2w}^{-j} \leq 1 | n_{1w}^{-i} = 5, w_{ij} = 2, n_{2w}^{-j} \leq 3) \approx 0.12^{16}$$

Therefore, in the case where $a_i^1 = a_j^1 = W$, $a_i^2 \neq a_j^2$ and $a_i^2 = W$ if $n_{1w}^{-i} = 5$ the optimal guess for agent i is $a_i^3 = W$. Using a similar argument as before, if $n_{1w}^{-i} \geq 5$ the optimal guess for agent i is $a_i^3 = W$. If $n_{1w}^{-i} \leq 2$, agent i would have agreed with agent j on B in the second round. Now

[15]

$$\frac{\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{3} 0.7^3 0.3^5 + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{3} 0.7^5 0.3^3}{\sum_{k=0}^{3} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

[16]

$$\frac{\sum_{k=0}^{1} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}{\sum_{k=0}^{3} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

consider the case where $n_{1w}^{-i} \in \{3,4\}$. In these cases, $a_i^3 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^3 = B$. That is, agent i switches to the second round guess of agent j if and only if $n_{1w}^{-i} \in \{3,4\}$.

Now we attend to the choice of agent j in the third round. We consider two cases. First, the case where $n_{2w}^{-j} = 3$. In this case, $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} \geq 5$ while $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 3$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \geq 5 | n_{2w}^{-j} = 3, w_{ij} = 2, n_{1w}^{-i} \geq 3) \approx 0.6^{17} \qquad p(n_{1w}^{-i} = 3 | n_{2w}^{-j} = 3, w_{ij} = 2, n_{1w}^{-i} \geq 3) \approx 0.21^{18}$$

Second, the case where $n_{2w}^{-j} = 2$. In this case, $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} \geq 6$ while $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} \leq 4$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \geq 6 | n_{2w}^{-j} = 2, w_{ij} = 2, n_{1w}^{-i} \geq 3) \approx 0.18^{19} \qquad p(n_{1w}^{-i} \leq 4 | n_{2w}^{-j} = 2, w_{ij} = 2, n_{1w}^{-i} \geq 3) \approx 0.67^{20}$$

Therefore, in the case where $a_i^1 = a_j^1 = W$, $a_i^2 \neq a_j^2$ and $a_i^2 = W$ if $n_{2w}^{-j} = 3$ the optimal guess for agent j is $a_j^3 = W$ while if $n_{2w}^{-j} = 2$ the optimal guess for agent j is $a_j^3 = B$. Using a similar argument as before, if $n_{2w}^{-i} \leq 2$ the optimal guess for agent j is $a_j^3 = B$. If $n_{2w}^{-j} \geq 4$, agent j would have agreed with agent i on W in the second round. That is, in the third round, agent j switches to the second round guess of agent i if and only if $n_{2w}^{-j} = 3$.

If both connectors switch, then there are either 8 or 9 w signals in the network. If $n_{1w}^{-i} = 3$ then agent i understands that there are exactly 8 w signals in the network and therefore $\forall t \geq 4 : a_i^t = B$. If $n_{1w}^{-i} = 4$ then agent i understands that there are exactly 9 w signals in the network and therefore $\forall t \geq 4 : a_i^t \in \{B, W\}$. Agent j cannot differentiate the two states unless she observes agent i guessing W at any period $t \geq 4$. That is, if $n_{1w}^{-i} = 3$ then $\forall t \geq 4 : a_j^t = B$ while if $n_{1w}^{-i} = 4$ then $\forall t_i^W \geq t \geq 4 : a_j^t = B$ and $\forall t > t_i^W : a_j^t \in \{B, W\}$ where t_i^W is the first period in which agent i

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$$\frac{\sum_{k=5}^{8}\left[\frac{1}{2} \times \binom{8}{3}0.7^3 0.3^5 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3}0.7^5 0.3^3 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8}\left[\frac{1}{2} \times \binom{8}{3}0.7^3 0.3^5 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3}0.7^5 0.3^3 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{8}{3}0.7^3 0.3^5 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{3}0.7^3 0.3^5 + \frac{1}{2} \times \binom{8}{3}0.7^5 0.3^3 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{3}0.7^5 0.3^3}{\sum_{k=3}^{8}\left[\frac{1}{2} \times \binom{8}{3}0.7^3 0.3^5 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{3}0.7^5 0.3^3 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=6}^{8}\left[\frac{1}{2} \times \binom{8}{2}0.7^2 0.3^6 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{2}0.7^6 0.3^2 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8}\left[\frac{1}{2} \times \binom{8}{2}0.7^2 0.3^6 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{2}0.7^6 0.3^2 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=3}^{4}\left[\frac{1}{2} \times \binom{8}{2}0.7^2 0.3^6 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{2}0.7^6 0.3^2 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8}\left[\frac{1}{2} \times \binom{8}{2}0.7^2 0.3^6 \times \binom{2}{2}0.7^2 0.3^0 \times \binom{8}{k}0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{2}0.7^6 0.3^2 \times \binom{2}{2}0.7^0 0.3^2 \times \binom{8}{k}0.7^{8-k} 0.3^k\right]}$$

guesses W starting from the fourth round ($t_i^W = \min\{t \geq 4 | a_i^t = W\}$).

If only agent i switches than it is clear that $n_{1w}^{-i} \leq 4$ while $n_{2w}^{-j} \leq 2$. That is, there are at most 8 w signals in the network. Hence, $\forall t \geq 4 : a_i^t = a_j^t = B$. If only agent j switches than it is clear that $n_{1w}^{-i} \geq 5$ while $n_{2w}^{-j} = 3$. That is, there are at least 10 w signals in the network. Hence, $\forall t \geq 4 : a_i^t = a_j^t = W$.

If no connector switches between the second and third rounds then at the beginning of the fourth round, agent i knows that since agent j guessed B in the second and third rounds it must be that $n_{2w}^{-j} \leq 2$. At the same time, agent j knows that since agent i guessed W in the second and third rounds it must be that $n_{1w}^{-i} \geq 5$.

To study the choice of agent i in the fourth round we begin with the case where $n_{1w}^{-i} = 6$. In this case, $a_i^4 = W$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 2$ while $a_i^4 = B$ is the unique optimal guess for agent i if $n_{2w}^{-j} = 0$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 2 | n_{1w}^{-i} = 6, w_{ij} = 2, n_{2w}^{-j} \leq 2) \approx 0.8^{21} \qquad p(n_{2w}^{-j} = 0 | n_{1w}^{-i} = 6, w_{ij} = 2, n_{2w}^{-j} \leq 2) \approx 0.03^{22}$$

Therefore, in the case where $a_i^1 = a_j^1 = W$, $a_i^2 = a_i^3 \neq a_j^3 = a_j^2$ and $a_i^2 = W$ if $n_{1w}^{-i} = 6$ the optimal guess for agent i in the fourth round is $a_i^4 = W$. Using a similar argument as before, if $n_{1w}^{-i} \geq 6$ the optimal guess for agent i is $a_i^4 = W$. If $n_{1w}^{-i} \leq 4$, agent i would have agreed with agent j on B in the second or third rounds (or use her tie breaking strategy). Now consider the case where $n_{1w}^{-i} = 5$. In this case, $a_i^4 = W$ is never the unique optimal guess for agent i. Therefore, it is optimal for agent i to guess $a_i^4 = B$. That is, in the fourth round, agent i switches to the second round guess of agent j if and only if $n_{1w}^{-i} = 5$.

Now we attend to the choice of agent j in the fourth round. We begin with the case where $n_{2w}^{-j} = 1$. In this case, $a_j^4 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} \geq 7$ while $a_j^4 = B$ is

[21]

$$\frac{\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{2} 0.7^2 0.3^6 + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{2} 0.7^6 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[22]

$$\frac{\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{0} 0.7^0 0.3^8 + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{2} 0.7^8 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

the unique optimal guess for agent j if $n_{1w}^{-i} = 5$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \geq 7 | n_{2w}^{-j} = 1, w_{ij} = 2, n_{1w}^{-i} \geq 5) \approx 0.12^{23} \qquad p(n_{1w}^{-i} = 5 | n_{2w}^{-j} = 1, w_{ij} = 2, n_{1w}^{-i} \geq 5) \approx 0.65^{24}$$

Therefore, in the case where $a_i^1 = a_j^1 = W$, $a_i^2 = a_i^3 \neq a_j^3 = a_j^2$ and $a_i^2 = W$ if $n_{2w}^{-j} = 1$ the optimal guess for agent j is $a_j^4 = B$. Using a similar argument as before, if $n_{2w}^{-j} \leq 1$ the optimal guess for agent j is $a_j^4 = B$. If $n_{2w}^{-j} \geq 3$, agent j would have agreed with agent i in the second or third rounds (or use her tie breaking strategy). Now consider the case where $n_{2w}^{-j} = 2$. In this case, $a_j^4 = B$ is never the unique optimal guess for agent j. Therefore, it is optimal for agent j to guess $a_j^4 = W$. That is, in the fourth round, agent j switches to the second round guess of agent i if and only if $n_{2w}^{-j} = 2$.

If both connectors switch, then there are 9 w signals in the network. Therefore $\forall t \geq 5 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. If only agent i switches than it is clear that $n_{1w}^{-i} \leq 5$ while $n_{2w}^{-j} \leq 1$. That is, there are at most 8 w signals in the network. Hence, $\forall t \geq 5 : a_i^t = a_j^t = B$. If only agent j switches than it is clear that $n_{1w}^{-i} \geq 6$ while $n_{2w}^{-j} = 2$. That is, there are at least 10 w signals in the network. Hence, $\forall t \geq 5 : a_i^t = a_j^t = W$.

If no connector switches between the third and fourth rounds then at the beginning of the fifth round, agent i knows that since agent j guessed B in the second, third and fourth rounds it must be that $n_{2w}^{-j} \leq 1$. At the same time, agent j knows that since agent i guessed W in the second, third and fourth rounds it must be that $n_{1r}^{-i} \geq 6$.

Note that in the fifth round if $n_{1w}^{-i} = 6$ then $a_i^5 = W$ is never the unique optimal guess for agent i while if $n_{1w}^{-i} \geq 7$ then $a_i^5 = B$ is never the unique optimal guess for agent i. Thus, agent i switches if and only if $n_{1w}^{-i} = 6$.

Now we attend to the choice of agent j in the fifth round. We begin with the case where $n_{2w}^{-j} = 0$. In this case, $a_j^5 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 8$ while $a_j^5 = B$ is the unique

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$$\frac{\sum_{k=7}^{8} \left[\frac{1}{2} \times \binom{8}{1} 0.7^1 0.3^7 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{1} 0.7^7 0.3^1 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{8}{1} 0.7^1 0.3^7 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{1} 0.7^7 0.3^1 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{8}{1} 0.7^1 0.3^7 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{5} 0.7^5 0.3^3 + \frac{1}{2} \times \binom{8}{1} 0.7^7 0.3^1 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{5} 0.7^3 0.3^5}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{8}{1} 0.7^1 0.3^7 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{1} 0.7^7 0.3^1 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

optimal guess for agent j if $n_{1w}^{-i} = 6$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} = 8 | n_{2w}^{-j} = 0, w_{ij} = 2, n_{1w}^{-i} \geq 6) \approx 0.03^{25} \qquad p(n_{1w}^{-i} = 6 | n_{2w}^{-j} = 0, w_{ij} = 2, n_{1w}^{-i} \geq 6) \approx 0.8^{26}$$

Therefore, in the case where $a_i^1 = a_j^1 = W$, $a_i^2 = a_i^3 = a_i^4 \neq a_j^4 = a_j^3 = a_j^2$ and $a_i^2 = W$ if $n_{2w}^{-j} = 0$ the optimal guess for agent j is $a_j^5 = B$. If $n_{2w}^{-j} \geq 2$, agent j would have agreed with agent i in the second, third or fourth rounds (or use her tie breaking strategy). Now consider the case where $n_{2w}^{-j} = 1$. In this case, $a_j^5 = B$ is never the unique optimal guess for agent j. Therefore, it is optimal for agent j to guess $a_j^5 = W$. That is, in the fifth round, agent j switches to the second round guess of agent i if and only if $n_{2w}^{-j} = 1$.

If both connectors switch, then there are 9 w signals in the network. Therefore $\forall t \geq 6 : a_i^t \in \{B, W\}, a_j^t \in \{B, W\}$. If only agent i switches than it is clear that $n_{1w}^{-i} = 6$ while $n_{2w}^{-j} = 0$. That is, there are 8 w signals in the network. Hence, $\forall t \geq 6 : a_i^t = a_j^t = B$. If only agent j switches than it is clear that $n_{1w}^{-i} \geq 7$ while $n_{2w}^{-j} = 1$. That is, there are at least 10 w signals in the network. Hence, $\forall t \geq 6 : a_i^t = a_j^t = W$. If no connector switches then it is clear that $n_{1w}^{-i} \geq 7$ while $n_{2w}^{-j} = 0$. That is, there are either 9 or 10 w signals in the network. If $n_{1w}^{-i} = 8$ then agent i understands that there are exactly 10 signals in the network and therefore $\forall t \geq 6 : a_i^t = W$. If $n_{1w}^{-i} = 7$ then agent i understands that there are exactly 9 signals in the network and therefore $\forall t \geq 6 : a_i^t \in \{B, W\}$. Agent j cannot differentiate the two states unless she observes agent i guessing B at any period $t \geq 6$. That is, if $n_{1w}^{-i} = 8$ then $\forall t \geq 6 : a_j^t = W$ while if $n_{1w}^{-i} = 7$ then $\forall t_i^B \geq t \geq 6 : a_j^t = W$ and $\forall t > t_i^B : a_j^t \in \{B, W\}$ where t_i^B is the first period in which agent i guesses B starting from the sixth round ($t_i^B = \min\{t \geq 6 | a_i^t = B\}$). This completes the proof of Result 7.7. □

C.8 Two Cores with One Link in the Naïve Model

Result 8. *Suppose G is a Two Cores with One Link network where n is even and $\frac{n}{2}$ is odd. Denote $\hat{\Delta}_k = |\{j \in N_k | s(j) = w\}| - |\{j \in N_k | s(j) = b\}|$ where $k \in \{1, 2\}$. By the naïve model:*

1. *$\forall h \in N$: If $s(h) = w$ then $a_h^1 = W$, otherwise, $a_h^1 = B$.*
2. *If $\hat{\Delta}_k \geq 1$ then $\forall h \in N_k, \forall t \geq 2 : a_h^t = W$ with two exceptions:*
    - *If $\hat{\Delta}_1 = 1$ and $a_j^1 = B$ then $a_i^2 \in \{B, W\}$.*
    - *If $\hat{\Delta}_2 = 1$ and $a_i^1 = B$ then $a_j^2 \in \{B, W\}$.*

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$$\frac{\frac{1}{2} \times \binom{8}{0} 0.7^0 0.3^8 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{8} 0.7^8 0.3^0 + \frac{1}{2} \times \binom{8}{0} 0.7^8 0.3^0 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{8} 0.7^0 0.3^8}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{8}{0} 0.7^0 0.3^8 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{0} 0.7^8 0.3^0 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

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$$\frac{\frac{1}{2} \times \binom{8}{0} 0.7^0 0.3^8 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{6} 0.7^6 0.3^2 + \frac{1}{2} \times \binom{8}{0} 0.7^8 0.3^0 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{6} 0.7^2 0.3^6}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{8}{0} 0.7^0 0.3^8 \times \binom{2}{2} 0.7^2 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{8}{0} 0.7^8 0.3^0 \times \binom{2}{2} 0.7^0 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

3. *If* $\hat{\Delta}_k \leq -1$ *then* $\forall h \in N_k, \forall t \geq 2 : a_h^t = B$ *with two exceptions:*

   - *If* $\hat{\Delta}_1 = -1$ *and* $a_j^1 = W$ *then* $a_i^2 \in \{B, W\}$.
   - *If* $\hat{\Delta}_2 = -1$ *and* $a_i^1 = W$ *then* $a_j^2 \in \{B, W\}$.

Proof. Definition 1 implies that each agent guesses by her own private signal in the first round. Note that every non-connector h has $b^{-C}(h) = 0$ while the connectors have $b^{-C}(i) = b^{-C}(j) = 1$. Also note that since $\frac{n}{2}$ is odd, $\hat{\Delta}_k \neq 0$. Therefore, if $\gamma_C = |\hat{\Delta}_k| > 1$, then, by Proposition 2, $\forall h \in N_k, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_k$ is positive and $a_h^t = B$ otherwise. In addition, if $\gamma_C = |\hat{\Delta}_k| = 1$ then, by Proposition 2, $\forall h \in N_k^{-m}, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_k$ is positive and $a_h^t = B$ otherwise, where $m = i$ if $k = 1$ and $m = j$ if $k = 2$. That is, we are left with the behavior of the connectors when $|\hat{\Delta}_k| = 1$. Suppose $k = 1$ and $\hat{\Delta}_1 = 1$. If $a_j^1 = W$ then agent i observes a majority of W in the second round and therefore, by Definition 1, guesses $a_i^2 = W$. However, if $a_j^1 = B$ then agent i observes a tie in the second round and therefore, by Definition 1, guesses $a_i^2 \in \{B, W\}$. Starting from the third round, she will observe at least $\frac{n}{2} - 1$ guesses of W out of $\frac{n}{2} + 1$ observations. Since $n > 2$ and even, this is a majority and $\forall t \geq 3 : a_i^t = W$. Similar reasoning applies for the cases where $k = 2$ or $\hat{\Delta}_1 = -1$. □

C.9 Two Cores with Three Links in the Bayesian Model

Let $N = N_1 \cup N_2$ where $|N_1| = \frac{n}{2}$ and $|N_2| = \frac{n}{2}$. Let agents $i_1, i_2, i_3 \in N_1$ and agent $j \in N_2$ be the four connectors, that is, $E \cap \{kl | k \in N_1, l \in N_2\} = \{i_1 j, i_2 j, i_3 j\}$. Denote $N_1^{-i} = N_1 \backslash \{i_1, i_2, i_3\}$, $N_1^{+j} = N_1 \cup \{j\}$, $N_2^{-j} = N_2 \backslash \{j\}$ and $N_2^{+i} = N_2 \cup \{i_1, i_2, i_3\}$. Denote the number of w signals in N_1, $|\{l \in N_1 | s(l) = w\}|$, by n_{1w}. Similarly, we denote the number of w signals in N_1^{-i}, N_1^{+j}, N_2, N_2^{-j} and N_2^{+i} by n_{1w}^{-i}, n_{1w}^{+j}, n_{2w}, n_{2w}^{-j} and n_{2w}^{+i}, respectively. We denote the number of w signals among the connectors by $w_{ij} = |\{k \in \{i_1, i_2, i_3, j\} | s(k) = w\}|$ (note that w_{ij} is known to agents i_1, i_2, i_3 and j). Finally, we denote the number of guesses of W in period t within the three connectors of N_1 by $W_i^t = |\{i \in \{i_1, i_2, i_3\} | a_t^i = W\}|$.

Result 9. *G is a Two Cores with Three Links network with $n = 18$ agents and the probability to receive a correct signal is $q = 0.7$. By the Bayesian model:*

1. $\forall k \in N$: *If* $s(k) = w$ *then* $a_k^1 = W$, *otherwise,* $a_k^1 = B$.
2. $\forall k \in N_1^{-i}$: *If* $n_{1w} > 4$ *then* $a_k^2 = W$, *otherwise,* $a_k^2 = B$. *In addition,* $\forall t > 2$: *If* $W_i^{t-1} \in \{0, 3\}$ *then* $a_k^t = a_{i_1}^{t-1}$, *otherwise* $a_k^t \in \{B, W\}$.
3. $\forall k \in N_2^{-j}$: *If* $n_{2w} > 4$ *then* $a_k^2 = W$, *otherwise,* $a_k^2 = B$. *In addition,* $\forall t > 2 : a_k^t = a_j^{t-1}$.
4. *Second round for connectors:*

   - $\forall i \in \{i_1, i_2, i_3\}$: *If* $n_{1w}^{+j} > 5$ *then* $a_i^2 = W$, *if* $n_{1w}^{+j} < 5$ *then* $a_i^2 = B$, *otherwise,* $a_i^2 \in \{B, W\}$.

- *If* $n_{2w}^{+i} > 6$ *then* $a_j^2 = W$*, if* $n_{2w}^{+i} < 6$ *then* $a_j^2 = B$*, otherwise,* $a_j^2 \in \{B, W\}$*.*

5. *Suppose* $W_i^2 \in \{1, 2\}$*. If* $n_{2w}^{-j} > 4$ *then* $\forall t > 2 : a_j^t = W$*, if* $n_{2w}^{-j} < 4$ *then* $\forall t > 2 : a_j^t = B$*, otherwise,* $\forall t > 2 : a_j^t \in \{B, W\}$*. In addition,* $\forall i \in \{i_1, i_2, i_3\}, \forall t > 2 : a_i^t = a_j^{t-1}$*.*

6. *If* $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2$ *then* $\forall t > 2$*:* $a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = a_j^2$*.*[27]

7. *Suppose the connectors disagree in the second round,* $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 \neq a_j^2$*. Denote* $K = n_{1w}^{+j}$ *and* $\bar{K} = n_{2w}^{+i}$*. Assume with no loss of generality that* $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = B$ *and* $a_j^2 = W$ *(*$K \leq 5$ *and* $\bar{K} \geq 6$*). Then,*

   (a) *Convergence to* W *occurs when* $K + \bar{K} - w_{ij} \geq 10$*, in the following scenarios:*

      i. *Immediate convergence: If one of the following conditions is satisfied:*

         A. $K \geq 3$ *and* $\bar{K} = 7$*.*

         B. $K = 5$ *and* $\bar{K} > 7$*.*

         C. $K = 4$*,* $\bar{K} \in \{8, 9\}$ *and* $w_{ij} \leq 1$*.*

         D. $K = 3$*,* $\bar{K} = 8$ *and* $w_{ij} = 0$*.*

         Then, $\forall t \geq 3 : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = W$*.*

      ii. *Mutual switch and convergence: If* $\bar{K} = 6$ *then,* $a_{i_1}^3 = a_{i_2}^3 = a_{i_3}^3 = W$*,* $a_j^3 = B$ *and* $\forall t \geq 4 : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = W$*.*

      iii. *Mutual keep and convergence: If one of the following conditions is satisfied:*

         A. $K = 2$ *and* $\bar{K} = 8$*.*

         B. $K = 4$ *and* $\bar{K} > 9$*.*

         C. $K = 4$*,* $\bar{K} \in \{8, 9\}$ *and* $w_{ij} \geq 2$*.*

         D. $K = 3$*,* $\bar{K} \in \{8, 9\}$ *and* $w_{ij} = 1$*.*

         Then, $a_{i_1}^3 = a_{i_2}^3 = a_{i_3}^3 = B$*,* $a_j^3 = W$ *and* $\forall t \geq 4 : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = W$*.*

      iv. *Double mutual keep and convergence: If one of the following conditions is satisfied:*

         A. $K = 2$ *and* $\bar{K} = 9$*.*

         B. $K = 3$ *and* $\bar{K} > 9$*.*

         C. $K = 3$*,* $\bar{K} = 9$ *and* $w_{ij} = 2$*.*

         Then, $\forall t \in \{3, 4\}$ *:*$a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $\forall t \geq 5 : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = W$*.*

      v. *Triple mutual keep and convergence: If* $K = 2$ *and* $\bar{K} = 10$ *then,* $\forall t \in \{3, 4, 5\}$ *:* $a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $\forall t \geq 6 : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = a_j^t = W$*.*

   (b) *Convergence to* B *occurs when* $K + \bar{K} - w_{ij} \leq 8$*, in the following scenarios:*

[27] A failure may occur in four cases where $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2$: (i) One subject in N_1^{-i}, two subjects in N_2^{-j} and all connectors recieve the same signal while all the others recieve the opposite signal , (ii) One subject in N_1^{-i}, three subjects in N_2^{-j} and all connectors recieve the same signal, (iii) Two subjects in N_1^{-i}, two subjects in N_2^{-j} and all connectors recieve the same signal, (iv) Two subjects in N_1^{-i}, three subjects in N_2^{-j} and three connectors recieve the same signal. The probability for a signal distribution that satisfies one of the four cases is 0.98%. Since we assume no tie breaking rule, 0.98% is an upper bound for the unconditional probability of such a failure.

i. *Immediate convergence: If one of the following conditions is satisfied:*

A. $K \leq 4$ *and* $\bar{K} = 6$.

B. $K = 4$ *and* $\bar{K} \in \{7, 8\}$.

C. $K = 3$, $\bar{K} = 7$ *and* $w_{ij} = 3$.

Then, $\forall t \geq 3 : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = a^t_j = B$.

ii. *Mutual switch and convergence: If* $K = 5$ *then,* $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = W$, $a^3_j = B$ *and* $\forall t \geq 4 : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = a^t_j = B$.

iii. *Mutual keep and convergence: If one of the following conditions is satisfied:*

A. $K = 3$ *and* $\bar{K} = 8$.

B. $K \leq 2$ *and* $\bar{K} = 7$.

C. $K = 3$, $\bar{K} = 7$ *and* $w_{ij} = 2$.

Then, $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = B$, $a^3_j = W$ *and* $\forall t \geq 4 : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = a^t_j = B$.

iv. *Double mutual keep and convergence: If* $K \leq 2$ *and* $\bar{K} = 8$ *then,* $\forall t \in \{3, 4\} : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = B, a^t_j = W$ *and* $\forall t \geq 5 : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = a^t_j = B$.

(c) *"Convergence" to indifference occurs when* $K + \bar{K} - w_{ij} = 9$, *in the following scenarios:*

i. *Mutual switch and "convergence": If* $K = 5$ *and* $\bar{K} = 6$ *then,* $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = W$, $a^3_j = B$ *and* $\forall t \geq 4 : a^t_{i_1}, a^t_{i_2}, a^t_{i_3}, a^t_j \in \{B, W\}$.

ii. *Mutual switch and two steps "convergence" type A: If* $\bar{K} = 6$ *and* $K \in \{3, 4\}$ *then,* $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = W$, $a^3_j = B$ *and* $\forall t \geq 4 : a^t_{i_1}, a^t_{i_2}, a^t_{i_3} \in \{B, W\}$. *Let* $\bar{t} = \min_{t \geq 4}\{a^t_{i_1} = B \quad OR \quad a^t_{i_2} = B \quad OR \quad a^t_{i_3} = B\}$. $\forall t \in \{4, \ldots, \bar{t}\} : a^t_j = W$ *and* $\forall t > \bar{t} : a^t_j \in \{B, W\}$.

iii. *Mutual switch and two steps "convergence" type B: If* $K = 5$ *and* $\bar{K} \in \{7, 8\}$ *then,* $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = W$, $a^3_j = B$ *and* $\forall t \geq 4 : a^t_j \in \{B, W\}$. *Let* $\bar{t} = \min_{t \geq 4}\{a^t_j = W\}$. $\forall t \in \{4, \ldots, \bar{t}\} : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = B$ *and* $\forall t > \bar{t} : a^t_{i_1}, a^t_{i_2}, a^t_{i_3} \in \{B, W\}$.

iv. *Mutual keep and two steps "convergence" type B: If* $K = 4$ *and* $\bar{K} = 9$ *then,* $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = B$, $a^3_j = W$ *and* $\forall t \geq 4 : a^t_j \in \{B, W\}$. *Let* $\bar{t} = \min_{t \geq 4}\{a^t_j = B\}$. $\forall t \in \{4, \ldots, \bar{t}\} : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = W$ *and* $\forall t > \bar{t} : a^t_{i_1}, a^t_{i_2}, a^t_{i_3} \in \{B, W\}$.

v. *Mutual keep, mutual switch and "convergence": If one of the following conditions is satisfied:*

A. $\bar{K} = 7$ *and* $K \in \{2, 3, 4\}$.

B. $\bar{K} = 8$ *and* $K = 4$.

Then, $a^3_{i_1} = a^3_{i_2} = a^3_{i_3} = B$, $a^3_j = W$ *and* $a^4_{i_1} = a^4_{i_2} = a^4_{i_3} = W$, $a^4_j = B$ *and* $\forall t \geq 5 : a^t_{i_1}, a^t_{i_2}, a^t_{i_3}, a^t_j \in \{B, W\}$.

vi. *Double keep and two steps "convergence" type A: If* $K = 1$ *and* $\bar{K} = 8$ *then,* $\forall t \in \{3, 4\} : a^t_{i_1} = a^t_{i_2} = a^t_{i_3} = B, a^t_j = W$ *and* $\forall t \geq 5 : a^t_{i_1}, a^t_{i_2}, a^t_{i_3} \in \{B, W\}$. *Let* $\bar{t} = \min_{t \geq 5}\{a^t_{i_1} = W \quad OR \quad a^t_{i_2} = W \quad OR \quad a^t_{i_3} = W\}$. $\forall t \in \{5, \ldots, \bar{t}\} : a^t_j = B$ *and* $\forall t > \bar{t} : a^t_j \in \{B, W\}$.

vii. *Double keep and two steps "convergence" type B: If* $K = 3$ *and* $\bar{K} = 9$ *then,* $\forall t \in \{3,4\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $\forall t \geq 5 : a_j^t \in \{B, W\}$*. Let* $\bar{t} = \min_{t \geq 5}\{a_j^t = B\}$*.* $\forall t \in \{5, \ldots, \bar{t}\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = W$ *and* $\forall t > \bar{t} : a_{i_1}^t, a_{i_2}^t, a_{i_3}^t \in \{B, W\}$*.*

viii. *Double keep, mutual switch and "convergence": If* $\bar{K} = 8$ *and* $K \in \{2, 3\}$ *then,* $\forall t \in \{3,4\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $a_{i_1}^5 = a_{i_2}^5 = a_{i_3}^5 = W$*,* $a_j^5 = B$ *and* $\forall t \geq 6 : a_{i_1}^t, a_{i_2}^t, a_{i_3}^t, a_j^t \in \{B, W\}$*.*

ix. *Triple keep and "convergence": If* $K = 1$ *and* $\bar{K} = 9$ *then,* $\forall t \in \{3,4,5\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $\forall t \geq 6 : a_{i_1}^t, a_{i_2}^t, a_{i_3}^t, a_j^t \in \{B, W\}$*.*

x. *Triple keep and two steps "convergence" type B: If* $K = 2$ *and* $\bar{K} = 9$ *then,* $\forall t \in \{3,4,5\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = B, a_j^t = W$ *and* $\forall t \geq 6 : a_j^t \in \{B, W\}$*. Let* $\bar{t} = \min_{t \geq 6}\{a_j^t = B\}$*.* $\forall t \in \{6, \ldots, \bar{t}\} : a_{i_1}^t = a_{i_2}^t = a_{i_3}^t = W$ *and* $\forall t > \bar{t} : a_{i_1}^t, a_{i_2}^t, a_{i_3}^t \in \{B, W\}$*.*

Proof. All agents are myopic Bayesian utility maximizers and that is a common knowledge. By Lemma 1 all agents are guessing their signal in the first period (result 9.1) and by Lemma 3 in the second period each agent chooses her action by the most popular signal in her local neighborhood (results 9.2, 9.3 and 9.4). By Proposition 1 the non-connectors of N_2 imitate the connector j (result 9.3).

The histories of agents i_1, i_2 and i_3 are identical. Denote these histories in period $t \geq 2$ by h_i^t. If $p(n_{1w} + n_{2w} \geq 9|h_i^t) > p(n_{1w} + n_{2w} \leq 9|h_i^t)$, it must be that $a_{i_1}^t = W$, $a_{i_2}^t = W$ and $a_{i_3}^t = W$, so that $W_i^t = 3$. However, if $p(n_{1w} + n_{2w} \geq 9|h_i^t) < p(n_{1w} + n_{2w} \leq 9|h_i^t)$, it must be that $a_{i_1}^t = B$, $a_{i_2}^t = B$ and $a_{i_3}^t = B$, so that $W_i^t = 0$. Therefore, if $W_i^t \in \{1, 2\}$ then it must be that $p(n_{1w} + n_{2w} \geq 9|h_i^t) = p(n_{1w} + n_{2w} \leq 9|h_i^t)$.

Since the histories of agents i_1, i_2 and i_3 are identical, each $k \in N_1^{-i}$ can consider agents i_1, i_2 and i_3 as one player denoted I. Then, by By Proposition 1, the optimal behavior of agent k is to imitate "agent" I starting from the third period. When $W_i^t \in \{0, 3\}$ then the guess of "agent" I is clear. In addition, since $W_i^t \in \{1, 2\}$ implies that "agent" I is indifferent, then agent k is also indifferent (result 9.2). Therefore, the dynamics is determined by the actions of "agent" I and agent j, the connectors, starting from $t = 3$.

Next, suppose that $W_i^2 \in \{1, 2\}$. Since the histories of agents i_1, i_2 and i_3 are identical then by result 9.4, it is clear for agent j that $n_{1w}^{+j} = 5$. Therefore, if $n_{2w}^{-j} \leq 3$ then $\forall t > 2 : a_j^t = B$, if $n_{2w}^{-j} \geq 5$ then $\forall t > 2 : a_j^t = W$ and if $n_{2w}^{-j} = 4$ then $\forall t > 2 : a_j^t \in \{B, W\}$. With no loss of generality assume that $a_j^2 = W$. At the beginning of the third round, agents i_1, i_2 and i_3 know $n_{1w}^{+j} = n_{1w}^{-i} + w_{ij} = 5$ and that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 6 - w_{ij}$. Thus, agents i_1, i_2 and i_3 attribute positive probability to the event that the total number of w signals is strictly lower than 9, when $w_{ij} \geq 3$ (recall that $w_{ij} \leq 4$). First, consider the case where $w_{ij} = 4$ (that is, $n_{1w}^{-i} = 1$). In this case, if $2 \leq n_{2w}^{-j} \leq 3$ then the unique optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 5$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} \leq 3 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.35^{28} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.5^{29}$$

Therefore, in the case where $w_{ij} = 4$, the optimal guess is $\forall i \in \{i_1, i_2, i_3\} : a_i^3 = W$. Consider the case where $w_{ij} = 3$ (that is, $n_{1w}^{-i} = 2$). In this case, if $n_{2w}^{-j} = 3$ then the unique optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 5$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.21^{30} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.6^{31}$$

Therefore, also in the case where $w_{ij} = 3$, the optimal guess is $\forall i \in \{i_1, i_2, i_3\} : a_i^3 = W$. Therefore, $\forall i \in \{i_1, i_2, i_3\} : a_i^3 = a_j^2$ independently of the value of w_{ij}.

In the fourth round, agents i_1, i_2 and i_3 know that agent j knows which signal is more frequent for sure. That is, when they observe a tie before the second period, and they know that agent j knows it (since $W_i^2 \in \{1, 2\}$), the optimal way for them to proceed is to imitate agent j starting from the third period, $\forall t > 2, \forall i \in \{i_1, i_2, i_3\} : a_i^t = a_j^{t-1}$ (result 9.5).

Our next step is to show that when all four connectors agree in the second round, that is, $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2$, they should stick to their second round guesses. With no loss of generality, let us consider the case where $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2 = W$.

We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 6 - w_{ij}$. Recall that agent i herself guessed W in the second round, therefore, $n_{1w}^{+j} \geq 5$. Thus, agent i may attribute positive probability to the event that the total number of w signals is lower than 9 if $w_{ij} = 4$ and $n_{1w}^{-i} \in \{1, 2\}$ or if $w_{ij} = 3$ and $n_{1w}^{-i} = 2$.

First, suppose that $w_{ij} = 4$ and $n_{1w}^{-i} = 1$, therefore $n_{2w}^{-j} \geq 2$. If $n_{2w}^{-j} \in \{2, 3\}$ then the unique

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$$\frac{\sum_{k=2}^{3} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{3} 0.7^3 0.3^5 + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{3} 0.7^5 0.3^3}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 5$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:[32]

$$p(n_{2w}^{-j} \leq 3 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.35^{33} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.5^{34}$$

Second, suppose $w_{ij} = 4$ and $n_{1w}^{-i} = 2$, therefore $n_{2w}^{-j} \geq 2$. If $n_{2w}^{-j} = 2$ then the unique optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 4$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:

$$p(n_{2w}^{-j} = 2 | n_{1w}^{-i} = 2, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.06^{35} \qquad p(n_{2w}^{-j} \geq 4 | n_{1w}^{-i} = 2, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.86^{36}$$

Finally, suppose $w_{ij} = 3$ and $n_{1w}^{-i} = 2$, therefore $n_{2w}^{-j} \geq 3$. If $n_{2w}^{-j} = 3$ then the unique optimal guess is $a_i^3 = B$ while if $n_{2w}^{-j} \geq 5$ then the unique optimal guess is $a_i^3 = W$. The conditional probabilities of these events are:[37]

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.21^{38} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.60^{39}$$

[32] This case is identical to the first case in the proof of result 9.5. In both cases the i's were indifferent. However, in the previous case they were not unanimous in their second round guess, while here they are. Since their individual guesses while indifferent bear no information, the calculations are identical.

[33]

$$\frac{\sum_{k=2}^{3}\left[\frac{1}{2}\times\binom{6}{1}0.7^1 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{1}0.7^5 0.3^1\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}{\sum_{k=2}^{8}\left[\frac{1}{2}\times\binom{6}{1}0.7^1 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{1}0.7^5 0.3^1\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[34]

$$\frac{\sum_{k=5}^{8}\left[\frac{1}{2}\times\binom{6}{1}0.7^1 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{1}0.7^5 0.3^1\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}{\sum_{k=2}^{8}\left[\frac{1}{2}\times\binom{6}{1}0.7^1 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{1}0.7^5 0.3^1\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[35]

$$\frac{\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{2}0.7^2 0.3^6+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{2}0.7^6 0.3^2}{\sum_{k=2}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[36]

$$\frac{\sum_{k=4}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}{\sum_{k=2}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{4}0.7^4 0.3^0\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[37] This case is identical to the second case in the proof of result 9.5. See footnote 32.

[38]

$$\frac{\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{3}0.7^3 0.3^1\times\binom{8}{3}0.7^3 0.3^5+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{3}0.7^1 0.3^3\times\binom{8}{3}0.7^5 0.3^3}{\sum_{k=3}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{3}0.7^3 0.3^1\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{3}0.7^1 0.3^3\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[39]

$$\frac{\sum_{k=5}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{3}0.7^3 0.3^1\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{3}0.7^1 0.3^3\times\binom{8}{k}0.7^{8-k}0.3^k\right]}{\sum_{k=3}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{3}0.7^3 0.3^1\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{3}0.7^1 0.3^3\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

Therefore, in the case where $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2 = W$, the optimal guess for every agent $i \in \{i_1, i_2, i_3\}$ is $a_i^3 = W$.

Now we move to the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agents $i \in \{i_1, i_2, i_3\}$ guessed W in the second round only if $n_{1w}^{-i} \geq 5 - w_{ij}$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{+i} \geq 6$. Thus, agent j may attribute positive probability to the event that the total number of w signals is lower than 9 if $w_{ij} = 4$ and $n_{2w}^{-j} \in \{2, 3\}$ or if $w_{ij} = 3$ and $n_{2w}^{-j} = 3$.

First, suppose $w_{ij} = 4$ and $n_{2w}^{-j} = 2$, therefore $n_{1w}^{-i} \geq 1$. If $n_{1w}^{-i} \in \{1, 2\}$ then the unique optimal guess is $a_j^3 = B$ while if $n_{1w}^{-i} \geq 4$ then the unique optimal guess is $a_j^3 = W$. The conditional probabilities of these events are:

$$p(n_{1w}^{-i} \leq 2 | n_{2w}^{-j} = 2, w_{ij} = 4, n_{1w}^{-i} \geq 1) \approx 0.37^{40} \qquad p(n_{1w}^{-i} \geq 4 | n_{2w}^{-j} = 2, w_{ij} = 4, n_{1w}^{-i} \geq 1) \approx 0.43^{41}$$

Second, suppose $w_{ij} = 4$ and $n_{2w}^{-j} = 3$, therefore $n_{1w}^{-i} \geq 1$. If $n_{1w}^{-i} = 1$ then the unique optimal guess is $a_j^3 = B$ while if $n_{1w}^{-i} \geq 3$ then the unique optimal guess is $a_j^3 = W$. The conditional probabilities of these events are:

$$p(n_{1w}^{-i} = 1 | n_{2w}^{-j} = 3, w_{ij} = 4, n_{1w}^{-i} \geq 1) \approx 0.06^{42} \qquad p(n_{1w}^{-i} \geq 3 | n_{2w}^{-j} = 3, w_{ij} = 4, n_{1w}^{-i} \geq 1) \approx 0.84^{43}$$

Finally, suppose $w_{ij} = 3$ and $n_{2w}^{-j} = 3$, therefore $n_{1w}^{-i} \geq 2$. If $n_{1w}^{-i} = 2$ then the unique optimal guess is $a_j^3 = B$ while if $n_{1w}^{-i} \geq 4$ then the unique optimal guess is $a_j^3 = W$. The conditional probabilities

[40]
$$\frac{\sum_{k=1}^{2}\left[\frac{1}{2}\times\binom{8}{2}0.7^2 0.3^6\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{2}0.7^6 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}{\sum_{k=1}^{6}\left[\frac{1}{2}\times\binom{8}{2}0.7^2 0.3^6\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{2}0.7^6 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

[41]
$$\frac{\sum_{k=4}^{6}\left[\frac{1}{2}\times\binom{8}{2}0.7^2 0.3^6\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{2}0.7^6 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}{\sum_{k=1}^{6}\left[\frac{1}{2}\times\binom{8}{2}0.7^2 0.3^6\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{2}0.7^6 0.3^2\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

[42]
$$\frac{\frac{1}{2}\times\binom{8}{3}0.7^3 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{1}0.7^1 0.3^5+\frac{1}{2}\times\binom{8}{3}0.7^5 0.3^3\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{1}0.7^5 0.3^1}{\sum_{k=1}^{6}\left[\frac{1}{2}\times\binom{8}{3}0.7^3 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{3}0.7^5 0.3^3\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

[43]
$$\frac{\sum_{k=3}^{6}\left[\frac{1}{2}\times\binom{8}{3}0.7^3 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{3}0.7^5 0.3^3\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}{\sum_{k=1}^{6}\left[\frac{1}{2}\times\binom{8}{3}0.7^3 0.3^5\times\binom{4}{4}0.7^4 0.3^0\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{3}0.7^5 0.3^3\times\binom{4}{4}0.7^0 0.3^4\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

of these events are:

$$p(n_{1w}^{-i} = 2 | n_{2w}^{-j} = 3, w_{ij} = 3, n_{1w}^{-i} \geq 2) \approx 0.24^{44} \qquad p(n_{1w}^{-i} \geq 4 | n_{2w}^{-j} = 3, w_{ij} = 3, n_{1w}^{-i} \geq 2) \approx 0.52^{45}$$

Therefore, in the case where $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = a_j^2 = W$, the optimal guess for agent j in the third round is $a_j^3 = W$.

The non-connectors are non-informative to the connectors starting from the second round (Lemma 5). That is, the third round provides no new information. Therefore, both will keep their guesses unchanged until the end of the game (result 9.6). There are four cases where myopic Bayesian utility maximizing agents will guess wrong:

1. $n_{1w}^{-i} = 1$, $n_{2w}^{-j} = 2$ and $w_{ij} = 4$.
2. $n_{1w}^{-i} = 1$, $n_{2w}^{-j} = 3$ and $w_{ij} = 4$.
3. $n_{1w}^{-i} = 2$, $n_{2w}^{-j} = 2$ and $w_{ij} = 4$.
4. $n_{1w}^{-i} = 2$, $n_{2w}^{-j} = 3$ and $w_{ij} = 3$.

The unconditional probability for these cases is approximately 0.489%. By symmetry, the probability for a similar distribution for the b signals is identical. Therefore, the probability for this case is approximately 0.978%. Since no prior on the tie breaking rule is assumed, we cannot provide an exact probability of failure.

Next, we study the case where in the second round the connectors in N_1 agree among themselves but disagree with the connector from N_2, that is, $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 \neq a_j^2$. With no loss of generality, let us consider the case where $a_{i_1}^2 = a_{i_2}^2 = a_{i_3}^2 = B$ while $a_j^2 = W$.

We first attend to the case where $w_{ij} = 0$. We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round, so that $n_{2w}^{-j} \geq 6$. Recall that agent i herself guessed B in the second round, therefore, $n_{1w}^{-i} \leq 5$. If $n_{1w}^{-i} \leq 1$, $a_i^3 = B$ is the optimal guess for agent $i \in \{i_1, i_2, i_3\}$ since there are at most 9 w signals. If $n_{1w}^{-i} \geq 3$, $a_i^3 = W$ is the optimal guess for agent $i \in \{i_1, i_2, i_3\}$ since there are at least 9 w signals. If $n_{1w}^{-i} = 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 6$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 8$. We compare

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$$\frac{\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{2} 0.7^2 0.3^4 + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{2} 0.7^4 0.3^2}{\sum_{k=2}^{6} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

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$$\frac{\sum_{k=4}^{6} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}{\sum_{k=2}^{6} \left[\frac{1}{2} \times \binom{8}{3} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{3} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

these two conditional probabilities:

$$p(n_{2w}^{-j} = 6 | n_{1w}^{-i} = 2, w_{ij} = 0, n_{2w}^{-j} \geq 6) \approx 0.80^{46} \qquad p(n_{2w}^{-j} = 8 | n_{1w}^{-i} = 2, w_{ij} = 0, n_{2w}^{-j} \geq 6) \approx 0.03^{47}$$

Therefore, if $n_{1w}^{-i} \leq 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} \geq 3$, $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$.

Let us now look into the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agent $i \in \{i_1, i_2, i_3\}$ guessed B in the second round only if $n_{1w}^{-i} \leq 5$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{-j} \geq 6$. If $n_{2w}^{-j} = 6$, $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} \leq 2$ while $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} \geq 4$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \leq 2 | n_{2w}^{-j} = 6, w_{ij} = 0, n_{1w}^{-i} \leq 5) \approx 0.43^{48} \qquad p(n_{1w}^{-i} \geq 4 | n_{2w}^{-j} = 6, w_{ij} = 0, n_{1w}^{-i} \leq 5) \approx 0.37^{49}$$

If $n_{2w}^{-j} = 7$, $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} \leq 1$ while $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} \geq 3$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \leq 1 | n_{2w}^{-j} = 7, w_{ij} = 0, n_{1w}^{-i} \leq 5) \approx 0.08^{50} \qquad p(n_{1w}^{-i} \geq 3 | n_{2w}^{-j} = 7, w_{ij} = 0, n_{1w}^{-i} \leq 5) \approx 0.81^{51}$$

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$$\frac{\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{8}{6} 0.7^6 0.3^2 + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{8}{6} 0.7^2 0.3^6}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

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$$\frac{\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{8}{8} 0.7^8 0.3^0 + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{8}{8} 0.7^0 0.3^8}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^{8-k} 0.3^k \right]}$$

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$$\frac{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}{\sum_{k=0}^{5} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

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$$\frac{\sum_{k=4}^{5} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}{\sum_{k=0}^{5} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

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$$\frac{\sum_{k=0}^{1} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}{\sum_{k=0}^{5} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

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$$\frac{\sum_{k=3}^{5} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}{\sum_{k=0}^{5} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

Therefore, if $n_{2w}^{-j} \geq 7$, $a_j^3 = W$ is the unique optimal guess for agent j while if $n_{2w}^{-j} = 6$, $a_i^3 = B$ is the unique optimal guess for agent j.

If agent $i \in \{i_1, i_2, i_3\}$ switched to $a_i^3 = W$ and agent j did not switch ($a_j^3 = W$), it means that there are at least 10 w signals and therefore $\forall t \geq 4 : a_i^t = W, a_j^t = W$. If agent $i \in \{i_1, i_2, i_3\}$ did not switch ($a_i^3 = B$) and agent j switched ($a_j^3 = B$), it means that there are at most 8 w signals and therefore $\forall t \geq 4 : a_i^t = B, a_j^t = B$. If both switched ($a_i^3 = W$ and $a_j^3 = B$) then it is clear that $n_{2w}^{-j} = 6$, therefore if $n_{1w}^{-i} \geq 4$, $\forall t \geq 4 : a_i^t = W, a_j^t = W$ while if $n_{1w}^{-i} = 3$, $\forall t \geq 4 : a_i^t \in \{B, W\}$ and if we denote $\bar{t}$ as the first $t \geq 4$ where $a_i^t = B$ for some $i \in \{i_1, i_2, i_3\}$ then $\forall \bar{t} \geq t \geq 4 : a_j^t = W$ $\forall t > \bar{t} : a_j^t \in \{B, W\}$.

If both did not switch, then both agent $i \in \{i_1, i_2, i_3\}$ and agent j know at the beginning of the fourth round that $w_{ij} = 0$, $n_{2w}^{-j} \geq 7$ and $n_{1w}^{-i} \leq 2$. We begin with the considerations of the connectors that belong to N_1. If $n_{1w}^{-i} = 2$ there are at least 9 w signals and therefore $a_i^4 = W$ is optimal for agent $i \in \{i_1, i_2, i_3\}$. Similarly, if $n_{1w}^{-i} \leq 1$ there are at most 9 w signals and therefore $a_i^4 = B$ is optimal for agent $i \in \{i_1, i_2, i_3\}$. Now we move to the considerations of the connector that belong to N_2. If $n_{2w}^{-j} = 7$ there are at most 9 w signals and therefore $a_j^4 = B$. However, if $n_{2w}^{-j} = 8$, $a_j^4 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 0$ while $a_j^4 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 2$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} = 0 | n_{2w}^{-j} = 8, w_{ij} = 0, n_{1w}^{-i} \leq 2) \approx 0.05^{52} \qquad p(n_{1w}^{-i} = 2 | n_{2w}^{-j} = 8, w_{ij} = 0, n_{1w}^{-i} \leq 2) \approx 0.74^{53}$$

Therefore, if $n_{2w}^{-j} = 8$, $a_j^4 = W$ is the unique optimal guess for agent j. If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^4 = W$, but agent j did not switch, $a_j^4 = W$ then $\forall t \geq 5 : a_i^t = W, a_j^t = W$ since there are 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^4 = B$, but agent j switched, $a_j^4 = B$ then $\forall t \geq 5 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. If both switched, $a_i^4 = W$ and $a_j^4 = B$, then $\forall t \geq 5 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals. If both did not switch ($a_i^4 = B$ and $a_j^4 = W$), then it is clear that $n_{2w}^{-j} = 8$, therefore if $n_{1w}^{-i} = 0$, $\forall t \geq 5 : a_i^t = B, a_j^t = B$ since there are 8 w signals. However, if $n_{1w}^{-i} = 1$, $\forall t \geq 5 : a_i^t \in \{B, W\}$ and if we denote $\hat{t}$ as the first $t \geq 5$ where $a_i^t = W$ then $\forall \hat{t} \geq t \geq 5 : a_j^t = B$ $\forall t > \hat{t} : a_j^t \in \{B, W\}$.

Next, consider the case where $w_{ij} = 1$. We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 5$. Recall that agent i herself guessed B in the second round, therefore, $n_{1w}^{-i} \leq 4$. If $n_{1w}^{-i} = 0$, $a_i^3 = B$ is the optimal guess for agent $i \in \{i_1, i_2, i_3\}$ since there are

[52]

$$\frac{\frac{1}{2} \times \binom{8}{8} 0.7^8 0.3^0 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{0} 0.7^0 0.3^6 + \frac{1}{2} \times \binom{8}{8} 0.7^0 0.3^8 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{0} 0.7^6 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{8} 0.7^8 0.3^0 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{8} 0.7^0 0.3^8 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

[53]

$$\frac{\frac{1}{2} \times \binom{8}{8} 0.7^8 0.3^0 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{2} 0.7^2 0.3^4 + \frac{1}{2} \times \binom{8}{8} 0.7^0 0.3^8 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{2} 0.7^4 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{8} 0.7^8 0.3^0 \times \binom{4}{0} 0.7^0 0.3^4 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{8} 0.7^0 0.3^8 \times \binom{4}{0} 0.7^4 0.3^0 \times \binom{6}{k} 0.7^{6-k} 0.3^k \right]}$$

at most 9 w signals. If $n_{1w}^{-i} \geq 3$, $a_i^3 = W$ is the optimal guess for agent $i \in \{i_1, i_2, i_3\}$ since there are at least 9 w signals. If $n_{1w}^{-i} = 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 5$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 7$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 5 | n_{1w}^{-i} = 2, w_{ij} = 1, n_{2w}^{-j} \geq 5) \approx 0.65^{54} \qquad p(n_{2w}^{-j} \geq 7 | n_{1w}^{-i} = 2, w_{ij} = 1, n_{2w}^{-j} \geq 5) \approx 0.12^{55}$$

Therefore, if $n_{1w}^{-i} \leq 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} \geq 3$, $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Let us now look into the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agent $i \in \{i_1, i_2, i_3\}$ guessed B in the second round only if $n_{1w}^{-i} \leq 4$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{-j} \geq 5$. If $n_{2w}^{-j} = 5$, $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} \leq 2$ while $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 4$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \leq 2 | n_{2w}^{-j} = 5, w_{ij} = 1, n_{1w}^{-i} \leq 4) \approx 0.52^{56} \qquad p(n_{1w}^{-i} = 4 | n_{2w}^{-j} = 5, w_{ij} = 1, n_{1w}^{-i} \leq 4) \approx 0.24^{57}$$

If $n_{2w}^{-j} = 6$, $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} \leq 1$ while $a_j^3 = W$ is the unique

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$$\frac{\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{5} 0.7^5 0.3^3 + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{5} 0.7^3 0.3^5}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=7}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

56

$$\frac{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

57

$$\frac{\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{4} 0.7^4 0.3^2 + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{4} 0.7^2 0.3^4}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

optimal guess for agent j if $n_{1w}^{-i} \geq 3$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} \leq 1 | n_{2w}^{-j} = 6, w_{ij} = 1, n_{1w}^{-i} \leq 4) \approx 0.12^{58} \qquad p(n_{1w}^{-i} \geq 3 | n_{2w}^{-j} = 6, w_{ij} = 1, n_{1w}^{-i} \leq 4) \approx 0.73^{59}$$

Therefore, if $n_{2w}^{-j} \geq 6$, $a_j^3 = W$ is the unique optimal guess for agent j while if $n_{2w}^{-j} = 5$, $a_i^3 = B$ is the unique optimal guess for agent j.

If agent $i \in \{i_1, i_2, i_3\}$ switched to $a_i^3 = W$ and agent j did not switch ($a_j^3 = W$), it means that there are at least 10 w signals and therefore $\forall t \geq 4 : a_i^t = W, a_j^t = W$. If agent $i \in \{i_1, i_2, i_3\}$ did not switch ($a_i^3 = B$) and agent j switched ($a_j^3 = B$), it means that there are at most 8 w signals and therefore $\forall t \geq 4 : a_i^t = B, a_j^t = B$. If both switched ($a_i^3 = W$ and $a_j^3 = B$) then it is clear that $n_{2w}^{-j} = 5$, therefore if $n_{1w}^{-i} = 4$, $\forall t \geq 4 : a_i^t = W, a_j^t = W$ while if $n_{1w}^{-i} = 3$, $\forall t \geq 4 : a_i^t \in \{B, W\}$ and if we denote $\bar{t}$ as the first $t \geq 4$ where $a_i^t = B$ for some $i \in \{i_1, i_2, i_3\}$ then $\forall \bar{t} \geq t \geq 4 : a_j^t = W$ $\forall t > \bar{t} : a_j^t \in \{B, W\}$.

If both did not switch, then both agent $i \in \{i_1, i_2, i_3\}$ and agent j know at the beginning of the fourth round that $w_{ij} = 1$, $n_{2w}^{-j} \geq 6$ and $n_{1w}^{-i} \leq 2$. First, note that if $n_{1w}^{-i} = 2$ there are at least 9 w signals and therefore $a_i^4 = W$ is optimal. Similarly, if $n_{1w}^{-i} = 0$ there are at most 9 w signals and therefore $a_i^4 = B$ is optimal. If $n_{1w}^{-i} = 1$, $a_i^4 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 6$ while $a_i^4 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 8$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 6 | n_{1w}^{-i} = 1, w_{ij} = 1, n_{2w}^{-j} \geq 6) \approx 0.80^{60} \qquad p(n_{2w}^{-j} = 8 | n_{1w}^{-i} = 1, w_{ij} = 1, n_{2w}^{-j} \geq 6) \approx 0.03^{61}$$

Therefore, if $n_{1w}^{-i} \leq 1$, $a_i^4 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 2$, $a_i^4 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Also, note that if $n_{2w}^{-j} = 6$ there are at most 9 w signals and therefore $a_j^4 = B$. However, if $n_{2w}^{-j} = 8$, there are at least 9 w signals and therefore $a_j^4 = W$. If $n_{2w}^{-j} = 7$ then $a_j^4 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 0$ while

[58]
$$\frac{\sum_{k=0}^{1} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

[59]
$$\frac{\sum_{k=3}^{4} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}{\sum_{k=0}^{4} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

[60]
$$\frac{\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{6} 0.7^6 0.3^2 + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{6} 0.7^2 0.3^6}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^3 0.3^1 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[61]
$$\frac{\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{8} 0.7^8 0.3^0 + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{8} 0.7^0 0.3^8}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^3 0.3^1 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

$a_j^4 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 2$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} = 0 | n_{2w}^{-j} = 7, w_{ij} = 1, n_{1w}^{-i} \leq 2) \approx 0.05^{62} \qquad p(n_{1w}^{-i} = 2 | n_{2w}^{-j} = 7, w_{ij} = 1, n_{1w}^{-i} \leq 2) \approx 0.74^{63}$$

Therefore, if $n_{2w}^{-j} \geq 7$, $a_j^4 = W$ is the unique optimal guess for agent j while if $n_{2w}^{-j} = 6$, $a_j^4 = B$ is the unique optimal guess for agent j.

If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^4 = W$, but agent j did not switch, $a_j^4 = W$ then $\forall t \geq 5 : a_i^t = W, a_j^t = W$ since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^4 = B$, but agent j switched, $a_j^4 = B$ then $\forall t \geq 5 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. If both switched, $a_i^4 = W$ and $a_j^4 = B$, then $\forall t \geq 5 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals.

If both did not switch ($a_i^4 = B$ and $a_j^4 = W$), then if $n_{1w}^{-i} = 0$ there are at most 9 w signals and therefore $a_i^5 = B$ is optimal while if $n_{1w}^{-i} = 1$ there are at least 9 w signals and therefore $a_i^5 = W$ is optimal. Similarly, if $n_{2w}^{-j} = 7$ there are at most 9 w signals and therefore $a_j^5 = B$ is optimal while if $n_{2w}^{-j} = 8$ there are at least 9 w signals and therefore $a_j^5 = W$ is optimal.

If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^5 = W$, but agent j did not switch, $a_j^5 = W$ then $\forall t \geq 6 : a_i^t = W, a_j^t = W$ since there are 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^5 = B$, but agent j switched, $a_j^5 = B$ then $\forall t \geq 6 : a_i^t = B, a_j^t = B$ since there are 8 w signals. If both switched, $a_i^5 = W$ and $a_j^5 = B$ or both did not switch, $a_i^5 = B$ and $a_j^5 = W$, then $\forall t \geq 6 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals.

Next, consider the case where $w_{ij} = 2$. We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 4$. Recall that agent i herself guessed B in the second round, therefore, $n_{1w}^{-i} \leq 3$. If $n_{1w}^{-i} = 3$, $a_i^3 = W$ is the optimal guess for agent $i \in \{i_1, i_2, i_3\}$ since there are at least 9 w signals. If $n_{1w}^{-i} = 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 4$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 6$. We compare

[62]
$$\frac{\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{0} 0.7^0 0.3^6 + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{0} 0.7^6 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

[63]
$$\frac{\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{2} 0.7^2 0.3^4 + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{2} 0.7^4 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{7} 0.7^7 0.3^1 \times \binom{4}{1} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{7} 0.7^1 0.3^7 \times \binom{4}{1} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

these two conditional probabilities:

$$p(n_{2w}^{-j}=4|n_{1w}^{-i}=2,w_{ij}=2,n_{2w}^{-j}\geq 4)\approx 0.44^{64} \qquad p(n_{2w}^{-j}\geq 6|n_{1w}^{-i}=2,w_{ij}=2,n_{2w}^{-j}\geq 4)\approx 0.31^{65}$$

Therefore, if $n_{1w}^{-i}\leq 2$, $a_i^3=B$ is the unique optimal guess for agent $i\in\{i_1,i_2,i_3\}$ while if $n_{1w}^{-i}=3$, $a_i^3=W$ is the unique optimal guess for agent $i\in\{i_1,i_2,i_3\}$. Let us now look into the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agent $i\in\{i_1,i_2,i_3\}$ guessed B in the second round only if $n_{1w}^{-i}\leq 3$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{-j}\geq 4$. If $n_{2w}^{-j}=4$ then there are at most 9 w signals, therefore $a_j^3=B$ is optimal. If $n_{2w}^{-j}=5$, $a_j^3=B$ is the unique optimal guess for agent j if $n_{1w}^{-i}\leq 1$ while $a_j^3=W$ is the unique optimal guess for agent j if $n_{1w}^{-i}=3$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i}\leq 1|n_{2w}^{-j}=5,w_{ij}=2,n_{1w}^{-i}\leq 3)\approx 0.21^{66} \qquad p(n_{1w}^{-i}=3|n_{2w}^{-j}=5,w_{ij}=2,n_{1w}^{-i}\leq 3)\approx 0.51^{67}$$

Therefore, if $n_{2w}^{-j}\geq 5$, $a_j^3=W$ is the unique optimal guess for agent j while if $n_{2w}^{-j}=4$, $a_i^3=B$ is the unique optimal guess for agent j.

If agent $i\in\{i_1,i_2,i_3\}$ switched to $a_i^3=W$ then $\forall t\geq 4: a_i^t=W, a_j^t=W$, since there are at least 10 w signals. If agent $i\in\{i_1,i_2,i_3\}$ did not switch, that is, $a_i^3=B$, but agent j switched to $a_j^3=B$ then $\forall t\geq 4: a_i^t=B, a_j^t=B$ since there are at most 8 w signals. If both switched ($a_i^3=W$ and $a_j^3=B$) then there are 9 w signals and as a result $\forall t\geq 4: a_i^t\in\{B,W\}, a_j^t\in\{B,W\}$. If both did not switch, then both agent $i\in\{i_1,i_2,i_3\}$ and agent j know at the beginning of the fourth round that $w_{ij}=2$, $n_{2w}^{-j}\geq 5$ and $n_{1w}^{-i}\leq 2$. First, note that if $n_{1w}^{-i}=2$ there are at least 9 w signals and therefore $a_i^4=W$ is optimal. If $n_{1w}^{-i}=1$, $a_i^4=B$ is the unique optimal guess for agent $i\in\{i_1,i_2,i_3\}$ if $n_{2w}^{-j}=5$ while $a_i^4=W$ is the unique optimal guess for agent $i\in\{i_1,i_2,i_3\}$ if $n_{2w}^{-j}\geq 7$. We compare

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[64]

$$\frac{\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{4}0.7^4 0.3^4+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{4}0.7^4 0.3^4}{\sum_{k=4}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[65]

$$\frac{\sum_{k=6}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^{8-k}0.3^k\right]}{\sum_{k=4}^{8}\left[\frac{1}{2}\times\binom{6}{2}0.7^2 0.3^4\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^k 0.3^{8-k}+\frac{1}{2}\times\binom{6}{2}0.7^4 0.3^2\times\binom{4}{2}0.7^2 0.3^2\times\binom{8}{k}0.7^{8-k}0.3^k\right]}$$

[66]

$$\frac{\sum_{k=0}^{1}\left[\frac{1}{2}\times\binom{8}{5}0.7^5 0.3^3\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{5}0.7^3 0.3^5\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^{6-k}0.3^k\right]}{\sum_{k=0}^{3}\left[\frac{1}{2}\times\binom{8}{5}0.7^5 0.3^3\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{5}0.7^3 0.3^5\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

[67]

$$\frac{\frac{1}{2}\times\binom{8}{5}0.7^5 0.3^3\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{3}0.7^3 0.3^3+\frac{1}{2}\times\binom{8}{5}0.7^3 0.3^5\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{3}0.7^3 0.3^3}{\sum_{k=0}^{3}\left[\frac{1}{2}\times\binom{8}{5}0.7^5 0.3^3\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^k 0.3^{6-k}+\frac{1}{2}\times\binom{8}{5}0.7^3 0.3^5\times\binom{4}{2}0.7^2 0.3^2\times\binom{6}{k}0.7^{6-k}0.3^k\right]}$$

these two conditional probabilities:

$$p(n_{2w}^{-j} = 5 | n_{1w}^{-i} = 1, w_{ij} = 2, n_{2w}^{-j} \geq 5) \approx 0.65^{68} \qquad p(n_{2w}^{-j} \geq 7 | n_{1w}^{-i} = 1, w_{ij} = 2, n_{2w}^{-j} \geq 5) \approx 0.12^{69}$$

Therefore, if $n_{1w}^{-i} \leq 1$, $a_i^4 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 2$, $a_i^4 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Also, note that if $n_{2w}^{-j} = 5$ there are at most 9 w signals and therefore $a_j^4 = B$. However, if $n_{2w}^{-j} \geq 7$, there are at least 9 w signals and therefore $a_j^4 = W$. If $n_{2w}^{-j} = 6$ then $a_j^4 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 0$ while $a_j^4 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 2$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} = 0 | n_{2w}^{-j} = 6, w_{ij} = 2, n_{1w}^{-i} \leq 2) \approx 0.05^{70} \qquad p(n_{1w}^{-i} = 2 | n_{2w}^{-j} = 6, w_{ij} = 2, n_{1w}^{-i} \leq 2) \approx 0.74^{71}$$

Therefore, if $n_{2w}^{-j} \geq 6$, $a_j^4 = W$ is the unique optimal guess for agent j while if $n_{2w}^{-j} = 5$, $a_j^4 = B$ is the unique optimal guess for agent j.

If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^4 = W$, but agent j did not switch, $a_j^4 = W$ then $\forall t \geq 5 : a_i^t = W, a_j^t = W$ since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^4 = B$, but agent j switched, $a_j^4 = B$ then $\forall t \geq 5 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. If both switched, $a_i^4 = W$ and $a_j^4 = B$, then $\forall t \geq 5 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals.

If both did not switch ($a_i^4 = B$ and $a_j^4 = W$), then if $n_{1w}^{-i} = 1$ there are at least 9 w signals and therefore $a_i^5 = W$ is optimal. However, if $n_{1w}^{-i} = 0$, $a_i^5 = B$ is uniquely optimal in case $n_{2w}^{-j} = 6$ while

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[68]
$$\frac{\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{5} 0.7^5 0.3^3 + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{5} 0.7^3 0.3^5}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[69]
$$\frac{\sum_{k=7}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[70]
$$\frac{\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{0} 0.7^0 0.3^6 + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{0} 0.7^6 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

[71]
$$\frac{\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{2} 0.7^2 0.3^4 + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{2} 0.7^4 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{6} 0.7^6 0.3^2 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{6} 0.7^2 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

$a_i^5 = W$ is uniquely optimal in case $n_{2w}^{-j} = 8$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 6 | n_{1w}^{-i} = 0, w_{ij} = 2, n_{2w}^{-i} \geq 6) \approx 0.80^{72} \qquad p(n_{2w}^{-j} = 8 | n_{1w}^{-i} = 0, w_{ij} = 2, n_{2w}^{-i} \geq 6) \approx 0.03^{73}$$

Therefore, if $n_{1w}^{-i} = 0$, $a_i^5 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 1$, $a_i^5 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Similarly, if $n_{2w}^{-j} \geq 7$ there are at least 9 w signals and therefore $a_j^5 = W$ is optimal while if $n_{2w}^{-j} = 6$ there are at most 9 w signals and therefore $a_j^5 = B$ is optimal.

If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^5 = W$, but agent j did not switch, $a_j^5 = W$ then $\forall t \geq 6 : a_i^t = W, a_j^t = W$ since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^5 = B$, but agent j switched, $a_j^5 = B$ then $\forall t \geq 6 : a_i^t = B, a_j^t = B$ since there are 8 w signals. If both switched, $a_i^5 = W$ and $a_j^5 = B$, then $\forall t \geq 6 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals. If both did not switch, that is $a_i^5 = B$ and $a_j^5 = W$, then it is clear that $n_{1w}^{-i} = 0$, therefore if $n_{2w}^{-j} = 8$, $\forall t \geq 6 : a_i^t = W, a_j^t = W$ since there are 10 w signals. However, if $n_{2w}^{-j} = 7$, $\forall t \geq 6 : a_j^t \in \{B, W\}$ and if we denote $\hat{t}$ as the first $t \geq 6$ where $a_j^t = B$ then $\forall \hat{t} \geq t \geq 6 : a_i^t = W$ and $\forall t > \hat{t} : a_i^t \in \{B, W\}$.

Next, consider the case where $w_{ij} = 3$. We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 3$. Recall that agent i herself guessed B in the second round, therefore, $n_{1w}^{-i} \leq 2$. If $n_{1w}^{-i} = 2$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 3$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 5$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 3 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.21^{74} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 2, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.60^{75}$$

If $n_{1w}^{-i} = 1$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \leq 4$ while $a_i^3 = W$

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$$\frac{\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{6} 0.7^6 0.3^2 + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{6} 0.7^2 0.3^6}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

73

$$\frac{\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{8} 0.7^8 0.3^0 + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{8} 0.7^0 0.3^8}{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{2} 0.7^2 0.3^2 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{3} 0.7^3 0.3^5 + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{3} 0.7^5 0.3^3}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

75

$$\frac{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{2} 0.7^2 0.3^4 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{2} 0.7^4 0.3^2 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 6$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} \leq 4 | n_{1w}^{-i} = 1, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.67^{76} \qquad p(n_{2w}^{-j} \geq 6 | n_{1w}^{-i} = 1, w_{ij} = 3, n_{2w}^{-j} \geq 3) \approx 0.18^{77}$$

Therefore, if $n_{1w}^{-i} \leq 1$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 2$, $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$.

Let us now look into the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agent $i \in \{i_1, i_2, i_3\}$ guessed B in the second round only if $n_{1w}^{-i} \leq 2$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{-j} \geq 3$. If $n_{2w}^{-j} \leq 4$ then there are at most 9 w signals, therefore $a_j^3 = B$ is optimal. If $n_{2w}^{-j} = 5$, $a_j^3 = B$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 0$ while $a_j^3 = W$ is the unique optimal guess for agent j if $n_{1w}^{-i} = 2$. We compare these two conditional probabilities:

$$p(n_{1w}^{-i} = 0 | n_{2w}^{-j} = 5, w_{ij} = 3, n_{1w}^{-i} \leq 2) \approx 0.05^{78} \qquad p(n_{1w}^{-i} = 2 | n_{2w}^{-j} = 5, w_{ij} = 3, n_{1w}^{-i} \leq 2) \approx 0.74^{79}$$

Therefore, if $n_{2w}^{-j} \geq 5$, $a_j^3 = W$ is the unique optimal guess for agent j while if $n_{2w}^{-j} \leq 4$, $a_j^3 = B$ is the unique optimal guess for agent j.

If agent $i \in \{i_1, i_2, i_3\}$ switched to $a_i^3 = W$ while agent j did not switch ($a_j^3 = W$), then $\forall t \geq 4 : a_i^t = W, a_j^t = W$, since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^3 = B$, but agent j switched to $a_j^3 = B$ then $\forall t \geq 4 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. If both switched ($a_i^3 = W$ and $a_j^3 = B$) then it is clear that $n_{1w}^{-i} = 2$, therefore if $n_{2w}^{-j} = 3$, $\forall t \geq 4 : a_i^t = B, a_j^t = B$ since there are 8 w signals. However, if $n_{2w}^{-j} = 4$, $\forall t \geq 4 : a_j^t \in \{B, W\}$ and if we denote $\bar{t}$ as the first $t \geq 4$ where $a_j^t = W$ then $\forall \bar{t} \geq t \geq 4 : a_i^t = B$ $\forall t > \bar{t} : a_i^t \in \{B, W\}$ for every $i \in \{i_1, i_2, i_3\}$.

If both did not switch, then both agent $i \in \{i_1, i_2, i_3\}$ and agent j know at the beginning of

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$$\frac{\sum_{k=3}^{4} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=3}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{0} 0.7^0 0.3^6 + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{0} 0.7^6 0.3^0}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

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$$\frac{\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{2} 0.7^2 0.3^4 + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{2} 0.7^4 0.3^2}{\sum_{k=0}^{2} \left[\frac{1}{2} \times \binom{8}{5} 0.7^5 0.3^3 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{6}{k} 0.7^k 0.3^{6-k} + \frac{1}{2} \times \binom{8}{5} 0.7^3 0.3^5 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{6}{k} 0.7^{6-k} 0.3^k\right]}$$

the fourth round that $w_{ij} = 3$, $n_{2w}^{-j} \geq 5$ and $n_{1w}^{-i} \leq 1$. First, note that if $n_{1w}^{-i} = 1$ there are at least 9 w signals and therefore $a_i^4 = W$ is optimal. If $n_{1w}^{-i} = 0$, $a_i^4 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} = 5$ while $a_i^4 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 7$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} = 5 | n_{1w}^{-i} = 0, w_{ij} = 3, n_{2w}^{-j} \geq 5) \approx 0.65^{80} \qquad p(n_{2w}^{-j} \geq 7 | n_{1w}^{-i} = 0, w_{ij} = 3, n_{2w}^{-j} \geq 5) \approx 0.12^{81}$$

Therefore, if $n_{1w}^{-i} = 0$, $a_i^4 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 1$, $a_i^4 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Also, note that if $n_{2w}^{-j} = 5$ there are at most 9 w signals and therefore $a_j^4 = B$. However, if $n_{2r}^{-j} \geq 6$, there are at least 9 w signals and therefore $a_j^4 = W$. If agent $i \in \{i_1, i_2, i_3\}$ switched, that is, $a_i^4 = W$, but agent j did not switch, $a_j^4 = W$ then $\forall t \geq 5 : a_i^t = W, a_j^t = W$ since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^4 = B$, but agent j switched, $a_j^4 = B$ then $\forall t \geq 5 : a_i^t = B, a_j^t = B$ since there are 8 w signals. If both switched, $a_i^4 = W$ and $a_j^4 = B$, then $\forall t \geq 5 : a_i^t \in \{W, B\}, a_j^t \in \{W, B\}$ since there are 9 w signals. If both did not switch ($a_i^4 = B$ and $a_j^4 = W$), then if $n_{2w}^{-j} \geq 7$ there are at least 10 w signals and therefore $\forall t \geq 5 : a_i^t = W, a_j^t = W$. However, if $n_{2w}^{-j} = 6$, $\forall t \geq 5 : a_j^t \in \{B, W\}$ and if we denote $\hat{t}$ as the first $t \geq 5$ where $a_j^t = B$ then $\forall \hat{t} \geq t \geq 5 : a_i^t = W$ $\forall t > \hat{t} : a_i^t \in \{B, W\}$.

Finally, consider the case where $w_{ij} = 4$. We begin with the considerations of the connectors that belong to N_1. At the beginning of the third round, agent $i \in \{i_1, i_2, i_3\}$ knows that agent j guessed W in the second round only if $n_{2w}^{-j} \geq 2$. Recall that agent i herself guessed B in the second round, therefore, $n_{1w}^{-i} \leq 1$. If $n_{1w}^{-i} = 1$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \leq 3$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 5$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} \leq 3 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.35^{82} \qquad p(n_{2w}^{-j} \geq 5 | n_{1w}^{-i} = 1, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.50^{83}$$

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$$\frac{\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{5} 0.7^5 0.3^3 + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{5} 0.7^3 0.3^5}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=7}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{3} 0.7^3 0.3^1 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{3} 0.7^1 0.3^3 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=2}^{3} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

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$$\frac{\sum_{k=5}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{1} 0.7^1 0.3^5 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{1} 0.7^5 0.3^1 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

If $n_{1w}^{-i} = 0$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \leq 4$ while $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ if $n_{2w}^{-j} \geq 6$. We compare these two conditional probabilities:

$$p(n_{2w}^{-j} \leq 4 | n_{1w}^{-i} = 0, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.78^{84} \qquad p(n_{2w}^{-j} \geq 6 | n_{1w}^{-i} = 0, w_{ij} = 4, n_{2w}^{-j} \geq 2) \approx 0.12^{85}$$

Therefore, if $n_{1w}^{-i} = 0$, $a_i^3 = B$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$ while if $n_{1w}^{-i} = 1$, $a_i^3 = W$ is the unique optimal guess for agent $i \in \{i_1, i_2, i_3\}$. Let us now look into the considerations of the connector that belongs to N_2. At the beginning of the third round, agent j knows that agent $i \in \{i_1, i_2, i_3\}$ guessed B in the second round only if $n_{1w}^{-i} \leq 1$. Recall that agent j herself guessed W in the second round, therefore, $n_{2w}^{-j} \geq 2$. If $n_{2w}^{-j} \leq 4$ then there are at most 9 w signals, therefore $a_j^3 = B$ is optimal. If $n_{2w}^{-j} \geq 5$ then there are at least 9 w signals, therefore $a_j^3 = W$ is optimal.

If agent $i \in \{i_1, i_2, i_3\}$ switched to $a_i^3 = W$ and j did not switch then $\forall t \geq 4 : a_i^t = W, a_j^t = W$, since there are at least 10 w signals. If agent $i \in \{i_1, i_2, i_3\}$ did not switch, that is, $a_i^3 = B$, but agent j switched to $a_j^3 = B$ then $\forall t \geq 4 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. If both switched ($a_i^3 = W$ and $a_j^3 = B$) then it is clear that $n_{1w}^{-i} = 1$, therefore if $n_{2w}^{-j} \leq 3$, $\forall t \geq 4 : a_i^t = B, a_j^t = B$ since there are at most 8 w signals. However, if $n_{2w}^{-j} = 4$, $\forall t \geq 4 : a_j^t \in \{B, W\}$ and if we denote $\bar{t}$ as the first $t \geq 4$ where $a_j^t = W$ then $\forall \bar{t} \geq t \geq 4 : a_i^t = B$ and $\forall t > \bar{t} : a_i^t \in \{B, W\}$. If both did not switch, ($a_i^3 = B$ and $a_j^3 = W$) then it is clear that $n_{1w}^{-i} = 0$, therefore if $n_{2w}^{-j} \geq 6$, $\forall t \geq 4 : a_i^t = W, a_j^t = W$ since there are at least 10 w signals. However, if $n_{2w}^{-j} = 5$, $\forall t \geq 4 : a_j^t \in \{B, W\}$ and if we denote $\hat{t}$ as the first $t \geq 4$ where $a_j^t = B$ then $\forall \hat{t} \geq t \geq 4 : a_i^t = W$ and $\forall t > \hat{t} : a_i^t \in \{B, W\}$. $\square$

C.10 Two Cores with Three Links in the Naïve Model

Result 10. *Suppose G is a Two Cores with Three Links network where $n = 18$. N_1 is the clique with the three connectors (i_1, i_2, i_3) and N_2 is the clique with the single connector (j). Denote $\hat{\Delta}_k = |\{j \in N_k | s(j) = w\}| - |\{j \in N_k | s(j) = b\}|$ where $k \in \{1, 2\}$ and $A_i = |\{i \in \{i_1 i_2, i_3\} | s(i) = w\}| - |\{i \in \{i_1 i_2, i_3\} | s(i) = b\}|$. By the naïve model:*

1. *$\forall h \in N$: If $s(h) = w$ then $a_h^1 = W$, otherwise, $a_h^1 = B$.*
2. *If $\hat{\Delta}_1 \geq 1$ then $\forall h \in N_1, \forall t \geq 2 : a_h^t = W$ with the exception that if $\hat{\Delta}_1 = 1$ and $a_j^1 = B$ then $a_{i_1}^2, a_{i_2}^2, a_{i_3}^2 \in \{B, W\}$.*

[84]
$$\frac{\sum_{k=2}^{4} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

[85]
$$\frac{\sum_{k=6}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}{\sum_{k=2}^{8} \left[\frac{1}{2} \times \binom{6}{0} 0.7^0 0.3^6 \times \binom{4}{4} 0.7^4 0.3^0 \times \binom{8}{k} 0.7^k 0.3^{8-k} + \frac{1}{2} \times \binom{6}{0} 0.7^6 0.3^0 \times \binom{4}{4} 0.7^0 0.3^4 \times \binom{8}{k} 0.7^{8-k} 0.3^k\right]}$$

3. *If* $\hat{\Delta}_1 \leq -1$ *then* $\forall h \in N_1, \forall t \geq 2 : a_h^t = B$ *with the exception that if* $\hat{\Delta}_1 = -1$ *and* $a_j^1 = W$ *then* $a_{i_1}^2, a_{i_2}^2, a_{i_3}^2 \in \{B, W\}$.

4. *If* $\hat{\Delta}_2 \geq 1$ *then* $\forall h \in N_2, \forall t \geq 2 : a_h^t = W$ *with the exception that if* $\hat{\Delta}_2 + A_i = 0$ *then* $a_j^2 \in \{B, W\}$ *and if* $\hat{\Delta}_2 + A_i < 0$ *then* $a_j^2 = B$.

5. *If* $\hat{\Delta}_2 \leq -1$ *then* $\forall h \in N_2, \forall t \geq 2 : a_h^t = B$ *with the exception that if* $\hat{\Delta}_2 + A_i = 0$ *then* $a_j^2 \in \{B, W\}$ *and if* $\hat{\Delta}_2 + A_i > 0$ *then* $a_j^2 = W$.

Proof. Definition 1 implies that each agent guesses by her own private signal in the first round. Note that every non-connector k has $b^{-C}(k) = 0$ while the connectors have $b^{-C}(i_1) = b^{-C}(i_2) = b^{-C}(i_3) = 1$ and $b^{-C}(j) = 3$. Also note that $\hat{\Delta}_1 \neq 0$ and $\hat{\Delta}_2 \neq 0$.

Therefore, if $|\hat{\Delta}_1| > 1$, then, by Proposition 2, $\forall h \in N_1, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_1$ is positive and $a_h^t = B$ otherwise. In addition, if $|\hat{\Delta}_1| = 1$ then, by Proposition 2, $\forall h \in N_1^{-i}, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_1$ is positive and $a_h^t = B$ otherwise. That is, we are left with the behavior of the connectors of N_1 when $|\hat{\Delta}_1| = 1$. Suppose that $\hat{\Delta}_1 = 1$. If $a_j^1 = W$ then agent $i \in \{i_1, i_2, i_3\}$ observes a majority of W in the second round and therefore, by Definition 1, guesses $a_i^2 = W$. However, if $a_j^1 = B$ then agent $i \in \{i_1, i_2, i_3\}$ observes a tie in the second round and therefore, by Definition 1, guesses $a_i^2 \in \{B, W\}$. Starting from the third round, they will observe at least 6 guesses of W out of 10 observations. This is a majority and $\forall t \geq 3 : a_i^t = W$. Similarly for negative $\hat{\Delta}_1$.

Now, we turn to the other clique, N_2. If $|\hat{\Delta}_2| > 3$, then, by Proposition 2, $\forall h \in N_2, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_2$ is positive and $a_h^t = B$ otherwise. In addition, if $|\hat{\Delta}_2| \leq 3$ then, by Proposition 2, $\forall h \in N_2^{-j}, \forall t \geq 2 : a_h^t = W$ if $\hat{\Delta}_2$ is positive and $a_h^t = B$ otherwise, since $|\hat{\Delta}_2| > 0$. That is, we are left with the behavior of the connector j when $|\hat{\Delta}_2| \in \{1, 3\}$. Suppose that $\hat{\Delta}_2$ is positive, that is, $\hat{\Delta}_2 \in \{1, 3\}$. If $\hat{\Delta}_2 + A_i > 0$ then agent j observes a majority of W in the second round and therefore, by Definition 1, guesses $a_j^2 = W$. However, if (i) $\hat{\Delta}_2 + A_i = 0$ then agent j observes a tie in the second round and therefore, by Definition 1, guesses $a_j^2 \in \{B, W\}$ or (ii) $\hat{\Delta}_2 + A_i < 0$ then agent j observes a majority of B in the second round and therefore, by Definition 1, guesses $a_j^2 = B$. Starting from the third round, agent j will observe at least 8 guesses of W out of 12 observations. This is a majority and $\forall t \geq 3 : a_j^t = W$. Similarly for negative $\hat{\Delta}_2$. □

C.11 Two Cores with One Link: Example

Consider the Two Cores with One Link network with 18 agents, divided into two cliques: $N_1 = \{i, i_1, \ldots, i_8\}$, where agent i is the connector, and $N_2 = \{j, j_1, \ldots, j_8\}$, where agent j is the connector.

Suppose that six agents in N_1, including connector i, receive a private signal of w, while only two agents in N_2, including connector j, receive the same signal. Thus, the correct guess is B.

At the end of round 1, agent i observes a total of seven guesses indicating w: her own, that of agent j, and five from agents in $N_1 \setminus \{i\}$. Agent j, in contrast, observes only three such guesses: his own, agent i's, and one from $N_2 \setminus \{j\}$. Therefore, in round 2, $a_i^2 = W$ and $a_j^2 = B$.

Importantly, for agent i to switch from W to B, she must believe that there is at most one w

signal in $N_2 \setminus \{j\}$—a set she cannot directly observe. Similarly, for agent j to switch from B to W, he would need to believe that there are at least seven w signals in $N_1 \setminus \{i\}$—which he also cannot observe.

At the end of round 2, agent i reasons as follows. If agent j had observed more than three w guesses in $N_2 \setminus \{j\}$, then a_j^2 would have been W. Therefore, agent i concludes that agent j observed at most three such guesses. Moreover, she compares the conditional probability that agent j saw exactly three agents with w signals to the conditional probability that he saw at most one. Since the former is more probable given her information, she maintains her current guess: $a_i^3 = W$. Similarly, agent j infers that agent i observed at least three w guesses in $N_1 \setminus \{i\}$—since otherwise, a_i^2 would have been B. He then considers whether agent i saw at most five such w signals, or at least seven. Since, from his perspective and given his information, the former is more likely, he also maintains his current guess: $a_j^3 = B$. Thus, neither connector switches between rounds 2 and 3.

However, information is being exchanged. By the end of round 2, agent i has already inferred that agent j observed at most three w guesses in $N_2 \setminus \{j\}$. In round 3, she reasons further: if agent j had observed exactly three such w guesses in $N_2 \setminus \{j\}$, he would have switched to $a_j^3 = W$. Since he does not switch and plays $a_j^3 = B$, she deduces that agent j observed at most two w guesses in $N_2 \setminus \{j\}$. Agent i now combines this inference with her own observation of seven w guesses in total. She concludes that there are at most nine w signals in the network. Since the total number of agents is 18, and signals are binary, she now determines that B is more likely and switches in round 4: $a_i^4 = B$. Meanwhile, agent j similarly infers from $a_i^3 = W$ that agent i must have observed at least five w guesses in $N_1 \setminus \{i\}$. He considers whether agent i saw exactly five such agents, or at least seven. From his perspective and given his information, the former is more probable. Therefore, he sticks with B: $a_j^4 = B$.

After round 4, it is now mutually understood that agent i observed five w guesses in $N_1 \setminus \{i\}$, and agent j observed at most one such guess in $N_2 \setminus \{j\}$. Together with their own signals and observations, both conclude that the majority of signals in the network are b. Accordingly, the connectors continue to play B in all subsequent rounds. The non-connectors, begin to imitate them starting in round 5.

D Rational Under Imitation?

D.1 Claim 1

Consider a *Single Aggregator* network with n participants. Assume:

1. n is even.

2. Every non-aggregator i has at most $\frac{n}{2} - 1$ direct neighbors, i.e., $b(i) < \frac{n}{2}$.

3. Every two non-aggregators i and j are either not linked, i.e., $ij \notin E$, or they share exactly the same set of neighbors, that is, $B(i) \backslash \{j\} = B(j) \backslash \{i\}$.

4. All subjects guess correctly in the first round.

5. The aggregator, denoted by A, never switches in the second round when her private signal coincides with the majority of first round guesses.

6. Agent A does not switch in the second round when her private signal coincides with the minority of first round guesses with probability $\alpha \in (0, 1]$.

7. Agent A does not switch in the second round when there is a tie in the first round guesses with probability $\beta \in [0, 1]$.

Claim 1 A Bayesian non-aggregator agent i imitates agent A if either (i) the aggregator switched between round 1 and round 2, i.e., $a_A^1 \neq a_A^2$, or (ii) the aggregator did not switch, and their initial guess was not in the first-round minority within agent i's local neighborhood, i.e., $a_A^1 = a_A^2$ and $|j \in B(i) \cup \{i\}| s(j) = s(A)| \geq |j \in B(i) \cup \{i\}| s(j) \neq s(A)|$. If the aggregator did not switch between round 1 and round 2 and their initial guess was in the first-round minority within agent i's local neighborhood, then there exist values of α and β for which imitation is not optimal for agent i.

Proof. By assumptions (iv) and (v) whenever the aggregator switches between round 1 and round 2, their second round guess is surely correct, therefore imitation is optimal. If the aggregator does not switch it might be that her private signal coincides with the majority of first round guesses or there is a tie (and then imitation is optimal) or, alternatively, that her private signal coincides with the minority of first round guesses and she decided not to switch. Therefore, when no switch is observed, a Bayesian non-aggregator agent i uses the $b(i) + 1$ first round guesses she observed and the fact that the aggregator did not switch, to evaluate the conditional probability that the aggregator's second round guess is incorrect.

With no loss of generality, assume that the aggregator recieved the private signal $s(A) = w$. By assumption (iv), $a_A^1 = W$. Therefore, agent i knows that the aggregator's signal is w. Let $m_w \in \{1, \ldots, b(i) + 1\}$ denote the number of W guesses observed by agent i at the end of the first period, including her own ($m_w = |\{j \in B(i) \cup \{i\}\}| a_j^1 = W|$). In addition, let $n_w \in \{1, \ldots, n\}$ be the number of W guesses observed by the aggregator A at the end of the first period ($n_w = |\{j \in$

$N|a_j^1 = W|)$. By property (iv), m_w and n_w denote also the number of w signals received by the neighbores of agents i and A (and themselves), respectively.

Hence, from the point of view of agent i before the second round, the aggregator will switch in probability $1-\alpha$ if w is the overall minority signal and in probability $1-\beta$ if there is a tie, formally, $(1-\alpha) \times P(n_w < \frac{n}{2}|m_w) + (1-\beta) \times P(n_w = \frac{n}{2}|m_w)$. The aggregator will not switch due to imitation-worthy reasons in probability 1 if w is the overall majority signal and in probability β if there is a tie, formally, $P(n_w > \frac{n}{2}|m_w) + \beta \times P(n_w = \frac{n}{2}|m_w)$. Finally, the aggregator, incorrectly, does not switch in probability α if w is the overall minority signal, formally, $\alpha \times P(n_w < \frac{n}{2}|m_w)$.

Note that by property (iii) the only new piece of information in the second round is whether the aggregator switched. If a switch was observed, agent i must imitate in the third round. If the aggregator did not switch, the probability that the aggregator's second round guess is incorrect, conditional on not switching is

$$\frac{\alpha \times P(n_w < \frac{n}{2}|m_w)}{P(n_w > \frac{n}{2}|m_w) + \beta \times P(n_w = \frac{n}{2}|m_w) + \alpha \times P(n_w < \frac{n}{2}|m_w)}$$

If this probability is greater than half, imitation is not optimal. That is, it is optimal for the non-aggregator not to imitate if and only if

$$\alpha \times P(n_w < \frac{n}{2}|m_w) > P(n_w > \frac{n}{2}|m_w) + \beta \times P(n_w = \frac{n}{2}|m_w)$$

Hence, it is optimal for the non-aggregator not to imitate if and only if

$$\alpha > \frac{P(n_w > \frac{n}{2}|m_w)}{P(n_w < \frac{n}{2}|m_w)} + \beta \frac{P(n_w = \frac{n}{2}|m_w)}{P(n_w < \frac{n}{2}|m_w)}$$

Note that,

$$P(n_w < \frac{n}{2}|m_w) = \frac{1}{2} \sum_{j=0}^{\frac{n}{2}-m_w-1} \binom{n-b(i)-1}{j} \left[q^j (1-q)^{n-b(i)-1-j} + q^{n-b(i)-1-j}(1-q)^j \right]$$

And,

$$P(n_w > \frac{n}{2}|m_w) = \frac{1}{2} \sum_{j=\frac{n}{2}-m_w+1}^{n-b(i)-1} \binom{n-b(i)-1}{j} \left[q^j (1-q)^{n-b(i)-1-j} + q^{n-b(i)-1-j}(1-q)^j \right] =$$

$$= \frac{1}{2} \sum_{j=0}^{n-b(i)-1-(\frac{n}{2}-m_w+1)} \binom{n-b(i)-1}{j} \left[q^j (1-q)^{n-b(i)-1-j} + q^{n-b(i)-1-j}(1-q)^j \right]$$

Hence, if $n-b(i)-1-(\frac{n}{2}-m_w+1) \geq \frac{n}{2}-m_w-1$ then $P(n_w < \frac{n}{2}|m_w) \leq P(n_w > \frac{n}{2}|m_w)$. Therefore, if $n-b(i)-1-(\frac{n}{2}-m_w+1) \geq \frac{n}{2}-m_w-1$ then $\frac{P(n_w > \frac{n}{2}|m_w)}{P(n_w < \frac{n}{2}|m_w)} \geq 1$.
So, if $n-b(i)-1-(\frac{n}{2}-m_w+1) \geq \frac{n}{2}-m_w-1$, since $\alpha \leq 1$, it is optimal for the non-aggregator to imitate. However, $n-b(i)-1-(\frac{n}{2}-m_w+1) \geq \frac{n}{2}-m_w-1$ if and only if $m_w \geq \frac{b(i)+1}{2}$. Therefore,

Network	Node's neighborhood	b(i)	Minority of Size 1	Minority of Size 2	Minority of Size 3	Minority of Size 4
Connected Spokes	Small Hub	3	$0.743 + 0.141\beta$	X	X	X
Connected Spokes	Large Hub	4	$0.574 + 0.17\beta$	$0.853 + 0.147\beta$	X	X
One Gatekeeper	Cluster	8	$0.022 + 0.087\beta$	$0.128 + 0.177\beta$	$0.375 + 0.231\beta$	$0.755 + 0.245\beta$

Table 1: Lower bound on α for imitation to be not optimal as a function of network position, minority size and β. The calculations follow the expressions in the proof of Claim 1.

$m_w \geq \frac{b(i)+1}{2}$ implies that imitation is optimal.

Finally, note that if $m_w < \frac{b(i)+1}{2}$ then $\frac{P(n_w > \frac{n}{2} | m_w)}{P(n_w < \frac{n}{2} | m_w)} < 1$. Therefore, there exist an $\alpha \in (0, 1]$ and β (e.g. $\beta = 0$) such that the condition holds and it is optimal for the non-aggregator not to imitate. □

D.2 Lower Bounds on Alpha

Following Claim 1 and its proof (Section D.1), we conclude that no under-imitation is expected by Bayesian subjects in the Star Network. However, non-aggregators in the Connected Spokes and in the One Gatekeeper networks may find it optimal not to imitate in cases where the aggregator does not switch between the first and the second rounds and her first round guess belongs to the local minority. Table 1 provides lower bounds on the rate of under-reaction to new information by the aggregators that is consistent with a decision by a Bayesian non-aggregator not to imitate as a function of its network position, the minority size and β (the rate of no switching by the aggregator in case of a tie).